

IB Demystified

Examiners · Moderators · Mentors

Mathematics: Analysis and Approaches Higher Level

Paper 1 — Mock Examination

Mock Exam 1

Non-calculator

Time allowed: **2 hours**

Maximum mark: **110 marks**

Candidate information

Candidate name:

Session number:

Date:

Topic emphasis: algebra, functions, calculus, trigonometry and vectors

Instructions to candidates

- Write your session number in the box on the cover.
- Do not begin this paper until you are told to do so.
- The use of any calculator is **not** permitted for this paper.
- **Section A:** answer every question. Record your answers in the boxes that follow each question; you may continue below the printed lines if you need more room.
- **Section B:** answer every question. Set out your solutions on separate answer-booklet pages, starting each question on a fresh page.
- Unless a question states otherwise, give numerical answers exactly, or rounded to three significant figures.
- Where a result is requested exactly, decimal approximations will not earn full marks.
- Show all of your reasoning. Marks may not be awarded for an answer that is not supported by working.
- A clean copy of the *Mathematics: Analysis and Approaches HL formula booklet* may be used.
- The maximum mark for this paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no supporting work. Where an answer is incorrect, marks may still be given for a valid method, provided the method is clearly shown. You are therefore strongly advised to show all working.

Section A

Answer **all** questions. Write your answers in the boxes provided. Working may be continued below the lines if necessary.

1.

[Maximum mark: 5]

The function f is defined by $f(x) = \frac{3x - 2}{x + 5}$, for $x \in \mathbb{R}$, $x \neq -5$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of $y = f(x)$;

(ii) the horizontal asymptote of the graph of $y = f(x)$.

[2]

(b) Find $f^{-1}(x)$, the inverse function of f .

[3]

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2.

[Maximum mark: 6]

Solve the equation $2 \cos^2 x + 3 \sin x - 3 = 0$, where $0 \leq x \leq 2\pi$.

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3.

[Maximum mark: 6]

A spherical balloon is being inflated so that its volume increases at a constant rate of $50 \text{ cm}^3 \text{ s}^{-1}$. At a particular instant the surface area of the balloon is $100\pi \text{ cm}^2$.

(a) Show that the radius of the balloon at this instant is 5 cm. [2]

(b) Find the rate at which the radius is increasing at this instant. [4]

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4.

[Maximum mark: 7]

Consider the binomial expansion of $(2 + ax)^5$ in ascending powers of x , where a is a positive constant.

- (a) Show that the coefficient of x^2 in this expansion is $80a^2$. [3]
- (b) Given that the coefficient of x^2 is 720, find the value of a . [2]
- (c) Hence find the coefficient of x^3 . [2]

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5.

[Maximum mark: 7]

Consider the equation $4^x - p2^x + 4 = 0$, where p is a real constant.

- (a) Show that, with the substitution $u = 2^x$, the equation can be written as $u^2 - pu + 4 = 0$. [2]
- (b) Hence find the set of values of p for which the original equation has two distinct real solutions. [5]

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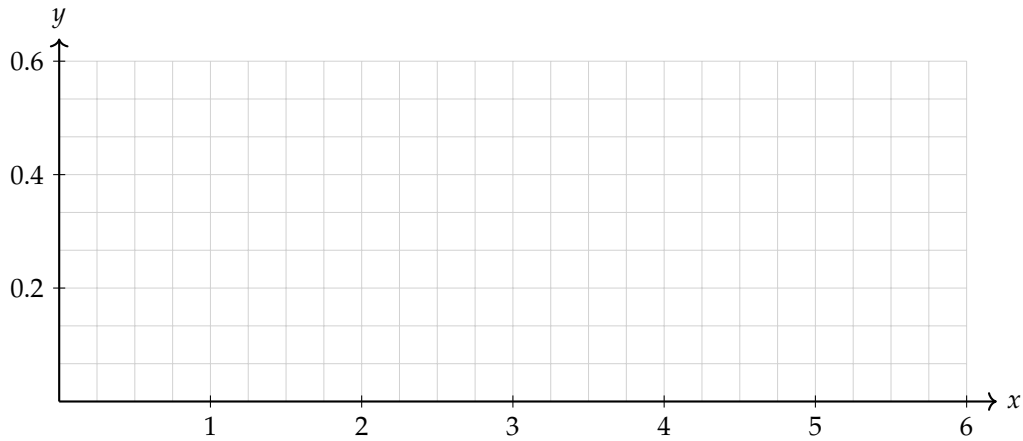
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6.

[Maximum mark: 8]

The function f is defined by $f(x) = x^2e^{-x}$, for $x \in \mathbb{R}$.

- (a) Find $f'(x)$. [3]
- (b) Show that the graph of $y = f(x)$ has stationary points at $x = 0$ and $x = 2$, and state the y -coordinate of each. [3]
- (c) On the axes below, sketch the graph of $y = f(x)$ for $0 \leq x \leq 7$, indicating the two stationary points. [2]



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Section B

Answer **all** questions in the answer booklet provided. Please begin each question on a new page.

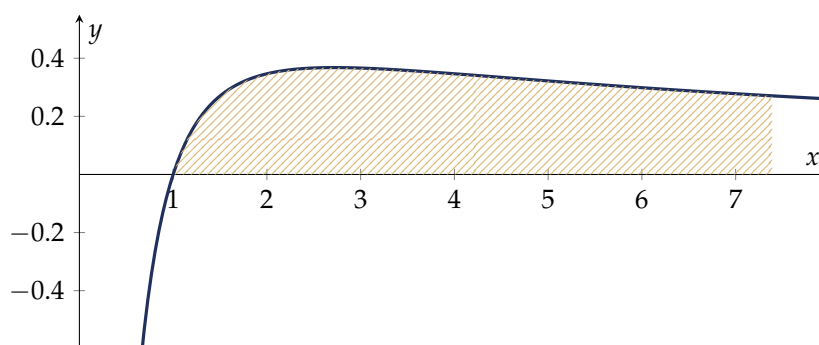
9. [Maximum mark: 12]

The sum of the first n terms of an arithmetic sequence is S_n , with first term a and common difference d . It is given that $S_4 = 36$ and $S_9 = 171$.

- (a) Show that $2a + 3d = 18$. [2]
- (b) Find the values of a and d . [4]
- (c) Write down an expression for the n th term u_n in terms of n . [2]
- (d) A geometric sequence has the same first term a and common ratio $r = \frac{1}{2}$. Find the sum to infinity of this geometric sequence. [2]
- (e) Find the least value of n for which $u_n > 100$. [2]

10. [Maximum mark: 13]

Consider the function $f(x) = \frac{\ln x}{x}$, for $x > 0$. Part of the graph of $y = f(x)$ is shown below.



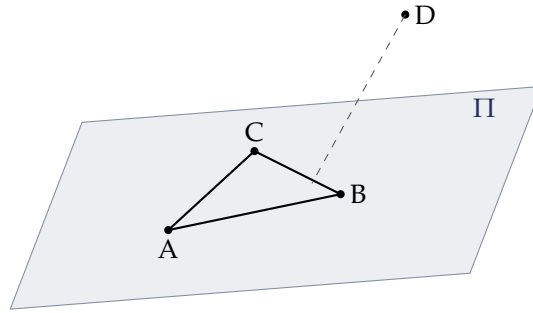
- (a) Find $f'(x)$. [3]
- (b) Show that the graph of $y = f(x)$ has exactly one stationary point and find its coordinates. [3]
- (c) Determine whether this stationary point is a maximum or a minimum. [2]
- (d) Find $\int \frac{\ln x}{x} dx$. [2]
- (e) The shaded region is bounded by the curve, the x -axis and the line $x = e^2$. Find the exact area of this region. [3]

11. [Maximum mark: 13]

- (a) Show that $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$. [3]
- (b) Hence solve $\frac{1 - \cos 2\theta}{\sin 2\theta} = \sqrt{3}$ for $0 < \theta < 2\pi$, $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$. [3]
- (c) The function h is defined by $h(\theta) = 3 \sin 2\theta + 4 \cos 2\theta$. Express $h(\theta)$ in the form $R \sin(2\theta + \varphi)$, where $R > 0$ and $0 < \varphi < \frac{\pi}{2}$. [4]
- (d) Hence write down the maximum value of $h(\theta)$ and find the smallest positive value of θ at which it occurs. [3]

12. [Maximum mark: 14]

The points $A(1, 2, 3)$, $B(3, 1, 4)$ and $C(2, -1, 5)$ are three vertices of a triangle. A plane Π contains A , B and C .



- (a) Find \vec{AB} and \vec{AC} . [2]
- (b) Find $\vec{AB} \times \vec{AC}$. [3]
- (c) Hence find the exact area of triangle ABC. [2]
- (d) Find the Cartesian equation of the plane Π . [3]
- (e) The point D has position vector $\begin{pmatrix} 5 \\ 0 \\ k \end{pmatrix}$. Given that the distance from D to Π is $\sqrt{35}$, find the possible values of k . [4]

End of examination.