IB Demystified

Examiners | Moderators | Mentors

Mathematics: Analysis and Approaches

Higher Level

Paper 3 — Mock Examination

Mock Examination 1

Calculator:	A graphic display calculator is **required**
Time allowed:	1 hour 15 minutes
Maximum mark:	55 marks

Information for candidates

Please read the following before you begin.

- A graphic display calculator is required throughout this paper.
- You may use a clean copy of the *Mathematics: Analysis and Approaches HL* formula booklet.
- Answer **both** questions. Begin each question on a fresh page of your answer booklet.
- Unless a question directs otherwise, give numerical answers either exactly or rounded to three significant figures.
- Any result obtained from your calculator must be accompanied by suitable supporting working. Where a graph is used to reach an answer, include a sketch or a clear description of the method.
- Answers given without sufficient working may not receive full marks.
- The total available for this paper is **55 marks**. The time allowed is **1 hour 15 minutes**.

Candidate name: . . . .

Session number: . . . .

Date: . . . .

This paper consists of two investigative questions. *Do not turn over until you are ready to begin.*

1. **[Maximum mark: 27]**

This question investigates how long a repeating experiment takes to produce its first success, and then compares this "waiting" process with a second reward scheme to decide which is the more consistent.

A single trial of an experiment succeeds with fixed probability p $(0 < p < 1)$, and trials are independent. Let the random variable X be the number of trials performed *up to and including* the first success.

(a) Take $p = 0.4$.

(i) Find $P(X = 2)$. **[2]**

(ii) Find $P(X \leq 3)$. **[2]**

(b) Show that, for a general value of p,

$$P(X = k) = p(1-p)^{k-1}, \qquad k \in \mathbb{Z}^+.$$

[2]

(c) The expected value of X is $E(X) = \sum_{k=1}^{\infty} k\, p(1-p)^{k-1}$.

Beginning from the geometric sum $\sum_{k=0}^{\infty} t^k = \frac{1}{1-t}$ for $|t| < 1$, differentiate with respect to t to establish a closed form for $\sum_{k=1}^{\infty} k\, t^{k-1}$, and hence show that

$$E(X) = \frac{1}{p}.$$

[5]

(d) With $p = 0.4$, consider the partial sums $T_N = \sum_{k=1}^{N} k\, p(1-p)^{k-1}$. Using your calculator, determine the least value of N for which T_N is within 0.001 of $E(X)$. **[4]**

(e) It can be shown that $\mathrm{Var}(X) = \frac{1-p}{p^2}$. Find the standard deviation of X when $p = 0.4$. **[2]**

A second scheme uses a random variable Y that records a player's score in a single round. The probability distribution of Y is partially given in the table below, where a is a constant.

y	1	2	3	4
$P(Y = y)$	0.2	a	0.3	0.2

(f) Find the value of a. **[3]**

(g) Show that $E(Y)$ is equal to $E(X)$ when $p = 0.4$. **[3]**

(h) Find $\mathrm{Var}(Y)$. **[2]**

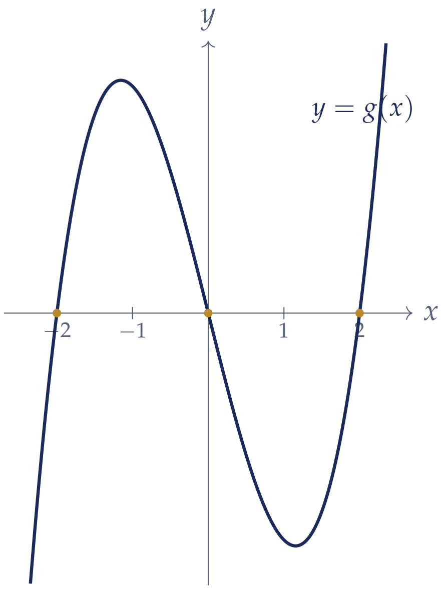
(i) The two schemes have the same expected value. State, with reasoning, which of X and Y describes the more *consistent* outcome. **[2]**

This question is worth 27 marks. Question 2 begins on the next page.

2. **[Maximum mark: 28]**

This question explores a family of cubic curves. Starting from a particular curve you will study its stationary points, its point of inflexion, and a tangent line; you will then generalise to obtain a tangent property and an enclosed-area result that does not depend on the curve chosen.

Consider the curve $g(x) = x^3 - 4x$.



The diagram shows $y = g(x)$. It is not drawn to scale.

(a) Find the coordinates of the stationary points of g, and determine the nature of each. **[4]**

(b) Find the coordinates of the point of inflexion of g. **[2]**

(c) Find the equation of the tangent to $y = g(x)$ at the point where $x = 1$, and show that this tangent meets the curve again. State the coordinates of the second point of intersection. **[5]**

(d) Using your calculator, sketch $y = g(x)$ together with the tangent from part (c) and confirm the coordinates of the second point of intersection. **[3]**

The curve is now generalised to the family

$$f(x) = x^3 + px + q,$$

where p and q are real constants. A tangent is drawn to $y = f(x)$ at the point where $x = t$.

(e) Show that this tangent meets the curve again at the point where $x = -2t$. **[5]**

(f) Let $L(x)$ be the tangent line in part (e). Show that

$$f(x) - L(x) = (x - t)^2 (x + 2t).$$

[4]

(g) Hence show that the area of the region enclosed between the curve $y = f(x)$ and the tangent line is

$$A = \frac{27\,t^4}{4},$$

and explain why this is the same for *every* curve in the family. **[5]**

End of Mock Examination 1.