IB Demystified

Examiners · Moderators · Mentors

Mathematics: Analysis and Approaches
Higher Level

Paper 1 — Mock Examination

Mock Exam 2

Non-calculator

Time allowed: **2 hours**

Maximum mark: **110 marks**

Candidate information

Candidate name: ....

Session number: ....

Date: ....

Topic emphasis: proof, complex numbers, calculus applications, functions and probability

Instructions to candidates

- Write your session number in the box on the cover.
- Do not begin this paper until you are told to do so.
- The use of any calculator is **not** permitted for this paper.
- **Section A:** answer every question. Record your answers in the boxes that follow each question; you may continue below the printed lines if you need more room.
- **Section B:** answer every question. Set out your solutions on separate answer-booklet pages, starting each question on a fresh page.
- Unless a question states otherwise, give numerical answers exactly, or rounded to three significant figures.
- Where a result is requested exactly, decimal approximations will not earn full marks.
- Show all of your reasoning. Marks may not be awarded for an answer that is not supported by working.
- A clean copy of the *Mathematics: Analysis and Approaches HL formula booklet* may be used.
- The maximum mark for this paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no supporting work. Where an answer is incorrect, marks may still be given for a valid method, provided the method is clearly shown. You are therefore strongly advised to show all working.

Section A

*Answer **all** questions. Write your answers in the boxes provided. Working may be continued below the lines if necessary.*

1. **[Maximum mark: 5]**

The functions f and g are defined by $f(x) = 2x - 3$ and $g(x) = \dfrac{x}{x+1}$, for $x \neq -1$.

(a) Find $(f \circ g)(x)$, giving your answer as a single fraction. **[3]**

(b) Hence solve $(f \circ g)(x) = 1$. **[2]**

2. **[Maximum mark: 6]**

Prove, by contradiction, that $\log_2 5$ is irrational.

3. **[Maximum mark: 6]**

Let $z = 1 - \mathrm{i}\sqrt{3}$.

(a) Find $|z|$ and $\arg z$, where $-\pi < \arg z \leq \pi$. **[3]**

(b) Hence find z^6, giving your answer in the form $a + b\mathrm{i}$. **[3]**

4. **[Maximum mark: 7]**

A closed cylindrical can has radius r cm, height h cm and a fixed volume of 250π cm^3. Let A cm^2 be its total surface area.

(a) Show that $A = 2\pi r^2 + \dfrac{500\pi}{r}$. **[3]**

(b) Find the value of r that minimizes A, justifying that this value gives a minimum. **[4]**

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5. **[Maximum mark: 7]**

A discrete random variable X has probability distribution given by

$$P(X = x) = kx, \qquad x = 1, 2, 3, 4,$$

where k is a constant.

(a) Find the value of k. **[2]**

(b) Find $E(X)$. **[3]**

(c) Find $P(X \geq 3)$. **[2]**

6. **[Maximum mark: 8]**

A curve is defined by the equation $x^2 + xy + y^2 = 7$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **[4]**

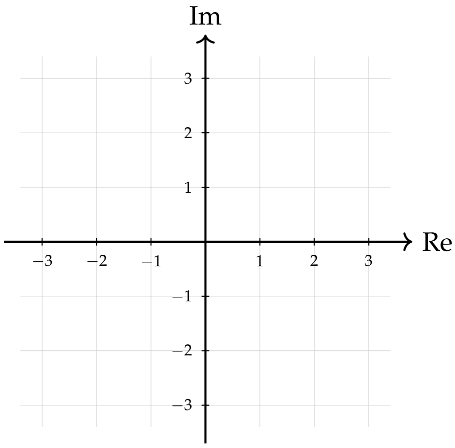
(b) Hence find the coordinates of the points on the curve at which the tangent is horizontal. **[4]**

7. **[Maximum mark: 9]**

(a) Express 8i in modulus–argument form. **[2]**

(b) Hence find the three roots of $z^3 = 8\mathrm{i}$, giving each in the form $r\mathrm{e}^{\mathrm{i}\theta}$ with $-\pi < \theta \leq \pi$. **[5]**

(c) Plot the three roots on the Argand diagram below and state the geometric shape that they form. **[2]**



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8. **[Maximum mark: 10]**

(a) Find $\int x \cos x \, \mathrm{d}x$. **[3]**

(b) Hence find $\int x^2 \sin x \, \mathrm{d}x$. **[4]**

(c) Evaluate $\int_0^{\pi} x^2 \sin x \, \mathrm{d}x$, giving your answer in exact form. **[3]**

Section B

*Answer **all** questions in the answer booklet provided. Please begin each question on a new page.*

9. **[Maximum mark: 12]**

(a) Prove by mathematical induction that

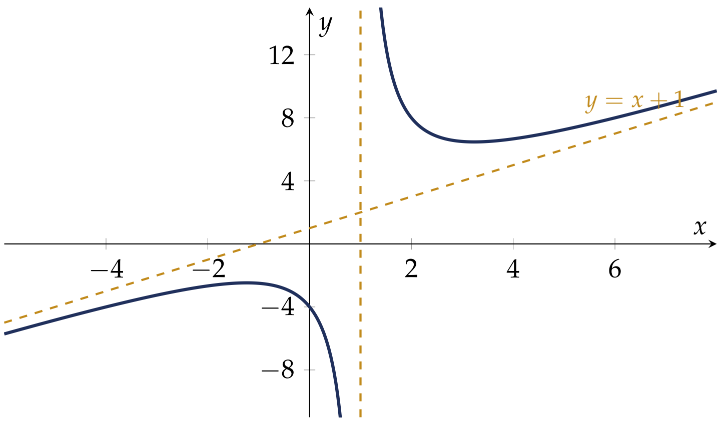
$$\sum_{r=1}^{n} r\,2^{r-1} = (n-1)2^n + 1 \qquad \text{for all } n \in \mathbb{Z}^+.$$

[9]

(b) Hence find the exact value of $\sum_{r=1}^{10} r\,2^{r-1}$. **[3]**

10. **[Maximum mark: 13]**

Consider the function $f(x) = \dfrac{x^2+4}{x-1}$, for $x \neq 1$. Part of the graph of $y = f(x)$ is shown below.



(a) Show that $f(x) = x + 1 + \dfrac{5}{x-1}$. **[3]**

(b) Write down the equations of the two asymptotes of $y = f(x)$. **[2]**

(c) Find $f'(x)$ and hence find the coordinates of the stationary points of $y = f(x)$. **[4]**

(d) Determine the nature of each stationary point. **[2]**

(e) Hence state the range of f. **[2]**

11. **[Maximum mark: 13]**

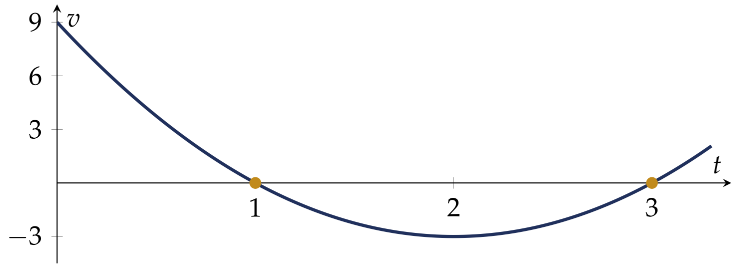
(a) Use De Moivre's theorem to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **[5]**

(b) Hence solve $8\cos^3\theta - 6\cos\theta = 1$ for $0 \leq \theta \leq \pi$. **[5]**

(c) Hence write down the three real roots of the equation $8t^3 - 6t - 1 = 0$, giving each in the form $\cos\left(\frac{a\pi}{9}\right)$. **[3]**

12. **[Maximum mark: 14]**

A particle moves in a straight line. Its velocity, v m s^{-1}, at time t seconds $(t \geq 0)$ is given by $v(t) = 3t^2 - 12t + 9$. The graph of v against t for $0 \leq t \leq 3$ is shown below.



(a) Find the times at which the particle is instantaneously at rest. **[2]**

(b) Find the acceleration of the particle when $t = 2$. **[2]**

(c) Given that the particle is at the origin when $t = 0$, find an expression for its displacement $s(t)$. **[3]**

(d) Find the total distance travelled by the particle during the first 3 seconds. **[5]**

(e) Find the maximum speed of the particle in the interval $0 \leq t \leq 3$. **[2]**

End of examination.