# IB Demystified

## Mathematics: Analysis and Approaches Higher Level

Paper 2 — Mock Examination

**Mock Exam 2 • Worked Solutions**

**Full worked solutions with mark allocations**

**GDC methods, progressive hints and examiner notes**

**Maximum mark: 110**

Mark abbreviations follow standard IB conventions: **M** method, **A** accuracy/answer, **R** reasoning, **AG** answer given, **FT** follow through. **GDC M**/**GDC A** denote correct calculator set-up and correct calculator output used appropriately.

IB Demystified — Examiners. Moderators. Mentors. — www.ibdemystified.com

# Section A

Solutions

## Question 1

*[7 marks]*

Exponential and logarithmic equations.

### Given

Three equations involving exponentials and logarithms.

### Formula / Law used

Logarithm of both sides; the law $\log a + \log b = \log(ab)$; GDC intersection for the mixed equation.

### Strategy

Take logs for (a); combine logs and solve a quadratic for (b); graph both sides for the transcendental equation in (c).

### Thought process

Parts (a) and (b) are solvable exactly; part (c) mixes an exponential with a linear term, so it needs a calculator graph and may have more than one root.

> **Hints**
>
> **Hint 1:** For (a), take the natural logarithm (or $\log_3$) of both sides.
> **Hint 2:** For (b), combine the two logarithms into one before removing the logarithm; reject any root outside the domain.
> **Hint 3:** For (c), graph $y_1 = 5e^{0.3x}$ and $y_2 = 2x + 40$ and use `intersect`; expect two solutions.

### Part (a)

$2x + 1 = \log_3 50 = \dfrac{\ln 50}{\ln 3}$ **M1**

$x = 1.28$ **A1**

> $x = 1.28$ (3 s.f.).

### Part (b)

$\log_2\big(x(x-2)\big) = 3 \;\Rightarrow\; x(x-2) = 2^3 = 8$ **M1**

$x^2 - 2x - 8 = 0 \;\Rightarrow\; (x-4)(x+2) = 0$

$x = 4$ or $x = -2$ **A1**

$x = -2$ is rejected since $\log_2(-2)$ is undefined **R1**

> $x = 4$.

### Part (c)

> **GDC Method**
>
> Graph $y_1 = 5e^{0.3x}$ and $y_2 = 2x + 40$. Use `intersect` (TI) / `G-Solv` → `INTSECT` (Casio); read both crossings.

intersections at $x = -20.0\ldots$ and $x = 8.06\ldots$ **GDC M1**

$x = -20.0$ or $x = 8.06$ **A1**

$x = -20.0$ or $x = 8.06$ (3 s.f.).

**Examiner note** For (b), always check the domain of each logarithm before accepting a root. For (c), there are two intersections — report both.

**Total: [7 marks]**

## Question 2

*[5 marks]*

Binomial distribution.

### Given

Free throws are independent with success probability $0.78$; $n = 12$ throws. Let $X \sim B(12, 0.78)$.

### Formula / Law used

Binomial p.m.f. and c.d.f.; expectation $E(X) = np$.

### Strategy

Direct p.m.f. for (a), complement of the cumulative for (b), the mean formula for (c).

### Thought process

"At least 10" is $P(X = 10) + P(X = 11) + P(X = 12)$, or equivalently $1 - P(X \leq 9)$.

**Hints**

**Hint 1:** Identify $n = 12$ and $p = 0.78$ before choosing a calculator command.
**Hint 2:** "At least 10" is most efficiently found as $1 - P(X \leq 9)$.
**Hint 3:** The expected value of a binomial random variable is $np$.

### Part (a)

**GDC Method**

`binompdf(12,0.78,10)`.

$$P(X = 10) = \binom{12}{10}(0.78)^{10}(0.22)^2$$ **GDC M1**

$$= 0.266\ldots$$ **A1**

$P(X = 10) = 0.266$ (3 s.f.).

### Part (b)

**GDC Method**

$1 -$ `binomcdf(12,0.78,9)`.

$$P(X \geq 10) = 1 - P(X \leq 9)$$ **M1**

$$= 0.489\ldots$$ **A1**

$P(X \geq 10) = 0.489$ (3 s.f.).

### Part (c)

$E(X) = np = 12 \times 0.78 = 9.36$ **A1**

Expected number of successful throws is $9.36$.

**Examiner note** The expectation need not be a whole number; do not round $9.36$ to an integer.

**Total: [5 marks]**

---

## Question 3

*[6 marks]*

Rational function: asymptotes, inverse and a fixed point.

### Given

$f(x) = \dfrac{3x+1}{x-2}$, $x \neq 2$.

### Formula / Law used

Vertical asymptote where the denominator is zero; horizontal asymptote from the ratio of leading coefficients; inverse by interchanging $x$ and $y$.

### Strategy

Read asymptotes from the form of $f$ (a); rearrange to invert (b); solve $f(x) = x$ as a quadratic (c).

### Thought process

For a rational function $\dfrac{ax+b}{cx+d}$, the horizontal asymptote is $y = a/c$ and the vertical asymptote is $x = -d/c$.

**Hints**

**Hint 1:** The vertical asymptote occurs where the denominator equals zero.
**Hint 2:** Compare the leading coefficients of numerator and denominator for the horizontal asymptote.
**Hint 3:** For the inverse, write $y = \dfrac{3x+1}{x-2}$, cross-multiply, and make $x$ the subject.

### Part (a)

vertical asymptote: $x = 2$ **A1**

horizontal asymptote: $y = 3$ **A1**

### Part (b)

$y(x-2) = 3x+1 \;\Rightarrow\; xy - 2y = 3x + 1$ **M1**

$x(y-3) = 2y+1$

$x = \dfrac{2y+1}{y-3}$ **A1**

$f^{-1}(x) = \dfrac{2x+1}{x-3}$ **A1**

### Part (c)

**GDC Method**

Solve $\dfrac{3x+1}{x-2} = x$, i.e. $x^2 - 5x - 1 = 0$, with the polynomial solver, or graph $y_1 = f(x)$ and $y_2 = x$ and use `intersect`.

$3x + 1 = x(x-2) \;\Rightarrow\; x^2 - 5x - 1 = 0$

$x = 5.19$ or $x = -0.193$ **A1**

> $x = 5.19$ or $x = -0.193$ (3 s.f.).

> **Examiner note** The inverse of a $\dfrac{ax+b}{cx+d}$ function is again of this type. A quick check: $f^{-1}$ should have its vertical asymptote at $y = 3$ (the old horizontal asymptote) — and it does, at $x = 3$.

**Total: [6 marks]**

---

## Question 4

*[6 marks]*

Bearings: cosine rule and sine rule.

### Given

$PA = 18$ km on bearing $052°$; $AB = 25$ km on bearing $124°$.

### Formula / Law used

Interior angle from bearings; cosine rule for $PB$; sine rule for the angle at $P$.

### Strategy

Find the interior angle $P\hat{A}B$, apply the cosine rule for $PB$, then the sine rule to find the angle $A\hat{P}B$ and hence the bearing.

### Thought process

The turn at $A$ changes the direction by $124° - 52° = 72°$, so the interior angle of the triangle at $A$ is $180° - 72° = 108°$.

> **Hints**
>
> **Hint 1:** Sketch the two legs and mark the bearings; the interior angle at $A$ is the supplement of the change in bearing.
> **Hint 2:** With two sides and the included angle, use the cosine rule for $PB$.
> **Hint 3:** The bearing of $B$ from $P$ is $052°$ plus the angle $A\hat{P}B$, found from the sine rule.

### Part (a)

$P\hat{A}B = 180° - (124° - 52°) = 108°$ **M1**

$PB^2 = 18^2 + 25^2 - 2(18)(25)\cos 108°$ **M1**

$PB = 35.0\ldots$ **A1**

> $PB = 35.0$ km (3 s.f.).

### Part (b)

$\dfrac{\sin(A\hat{P}B)}{25} = \dfrac{\sin 108°}{35.03\ldots}$ **M1**

$A\hat{P}B = 42.7\ldots°$ **A1**

bearing $= 052° + 42.7° = 094.7°$ **A1**

> The bearing of $B$ from $P$ is $094.7°$ (or $095°$ to the nearest degree).

**Examiner note** Bearings are measured clockwise from north and written with three figures (e.g. $094.7°$). Use the stored value of $PB$, not the rounded $35.0$, in the sine rule.

**Total: [6 marks]**

---

## Question 5

*[7 marks]*

Exponential (non-linear) regression.

### Given

Six data points $(t, N)$; model $N = a\,b^t$.

### Formula / Law used

Exponential regression on the GDC; using the model for prediction and for solving $N = 2000$.

### Strategy

Enter the data and run exponential regression for $a$, $b$ and $r$ (a); substitute $t = 7$ (b); solve $a\,b^t = 2000$ (c).

### Thought process

The counts roughly multiply by a constant factor each hour, which is the signature of exponential growth, so exponential regression is appropriate.

> **Hints**
>
> **Hint 1:** Enter $t$ in one list and $N$ in another, then choose *exponential* regression.
> **Hint 2:** Read $a$, $b$ and the correlation coefficient directly from the regression output.
> **Hint 3:** For (c), solve $a\,b^t = 2000$ by graphing or by taking logarithms.

### Part (a)

> **GDC Method**
>
> List 1 $= t$, List 2 $= N$. Run *exponential regression* $y = a\,b^x$: `ExpReg` (TI) or `CALC` $\to$ `ab Exp` (Casio).

$a = 49.4\ldots$ **GDC A1**

$b = 1.58\ldots$ **GDC A1**

$r = 1.00 \quad (0.99993\ldots)$ **A1**

> $N = 49.4\,(1.58)^t$, with $r = 1.00$ (3 s.f.).

### Part (b)

> **GDC Method**
>
> Evaluate the stored regression equation at $x = 7$, or use the table feature.

$N = 49.43\ldots \times (1.5795\ldots)^7$ **M1**

$= 1210\ldots$ **A1**

Estimated count after 7 hours is about $1210$ bacteria (3 s.f.).

### Part (c)

**GDC Method**

Graph $y_1 = 49.43(1.5795)^x$ and $y_2 = 2000$; use `intersect`.

$49.43\ldots(1.5795\ldots)^t = 2000$ **M1**

$t = 8.10\ldots$ **A1**

The count first exceeds 2000 after $t = 8.10$ hours (3 s.f.).

**Examiner note** Carry the unrounded $a$ and $b$ into parts (b) and (c). Using $a = 49.4$, $b = 1.58$ can shift the third significant figure.

**Total: [7 marks]**

## Question 6

*[6 marks]*

Normal distribution including a conditional probability.

### Given

Lifetime $X \sim N(8200,\, 540^2)$ hours.

### Formula / Law used

Normal CDF; conditional probability $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

### Strategy

Direct probabilities for (a)–(b); for (c) note that $\{X > 9000\} \subset \{X > 8000\}$.

### Thought process

Since lasting more than $9000$ already implies lasting more than $8000$, the intersection in (c) is just $P(X > 9000)$.

> **Hints**
>
> **Hint 1:** Identify mean and standard deviation, then decide on the bounds for each probability.
> **Hint 2:** For (b) use normal CDF with lower bound $7500$ and upper bound $8500$.
> **Hint 3:** For (c), the event $X > 9000$ is contained in $X > 8000$, so divide $P(X > 9000)$ by $P(X > 8000)$.

### Part (a)

> **GDC Method**
>
> `normalcdf(9000,1E99,8200,540)`.

$P(X > 9000)$ **GDC M1**

$= 0.0692\ldots$ **A1**

$P(X > 9000) = 0.0692$ (3 s.f.).

### Part (b)

$P(7500 < X < 8500) = \texttt{normalcdf}(7500, 8500, 8200, 540)$ **GDC M1**

$= 0.613\ldots$ **A1**

$P(7500 < X < 8500) = 0.613$ (3 s.f.).

### Part (c)

$P(X > 9000 \mid X > 8000) = \dfrac{P(X > 9000)}{P(X > 8000)} = \dfrac{0.06923\ldots}{0.64432\ldots}$ **M1**

$= 0.107\ldots$ **A1**

$P(X > 9000 \mid X > 8000) = 0.107$ (3 s.f.).

> **Examiner note** Recognising that one event is contained in the other avoids any need to compute an intersection separately.

**Total: [6 marks]**

## Question 7

*[7 marks]*

Area under a curve and area between a curve and a line.

### Given

$y = \dfrac{8}{x^2+1}$ and the line $y = 2$.

### Formula / Law used

Definite integral for area; intersection of curve and line; numerical integration on the GDC.

### Strategy

Numerical integral from $0$ to $3$ for (a). For (b), find where $\dfrac{8}{x^2+1} = 2$, then integrate the difference between curve and line over that interval.

### Thought process

For (b) the curve lies above the line between the intersection points, so the enclosed area is $\int(\text{curve} - \text{line})\,dx$.

> **Hints**
>
> **Hint 1:** Area under a curve above the $x$-axis is a definite integral; use numerical integration.
>
> **Hint 2:** For (b), first solve $\dfrac{8}{x^2+1} = 2$ to find the limits of integration.
>
> **Hint 3:** Integrate (top curve − bottom line) between the two intersection $x$-values.

### Part (a)

> **GDC Method**
>
> Compute $\displaystyle\int_0^3 \frac{8}{x^2+1}\,dx$ with the numerical integration template.

Area $= \displaystyle\int_0^3 \frac{8}{x^2+1}\,dx$ **M1**

numerical integration **GDC M1**

$= 9.99\ldots$ **A1**

> Area $= 9.99$ (3 s.f.).

### Part (b)

$\dfrac{8}{x^2+1} = 2 \;\Rightarrow\; x^2 + 1 = 4$ **M1**

$x = \pm\sqrt{3}$ **A1**

> **GDC Method**
>
> Compute $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} \left(\frac{8}{x^2+1} - 2\right) dx$.

Area $= \displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} \left(\frac{8}{x^2+1} - 2\right) dx$ **M1**

$= 9.82\ldots$ **A1**

Enclosed area $= 9.83$ (3 s.f.).

**Examiner note** Subtract the lower function from the upper one. Forgetting to subtract the line $y = 2$ gives the area under the curve instead of the enclosed region.

**Total: [7 marks]**

## Question 8

*[7 marks]*

Separable differential equation.

### Given

$\frac{dy}{dx} = \frac{2x}{y}$, $y > 0$, with $y(1) = 3$.

### Formula / Law used

Separation of variables; integrate both sides; apply the initial condition.

### Strategy

Separate, integrate, find the constant, then express $y$ explicitly.

### Thought process

The variables separate cleanly: $y\,dy = 2x\,dx$. The condition $y > 0$ selects the positive square root at the end.

> **Hints**
>
> **Hint 1:** Rearrange so all $y$ terms are with $dy$ and all $x$ terms with $dx$.
> **Hint 2:** Integrate both sides and include a single constant of integration.
> **Hint 3:** Use $y(1) = 3$ to find the constant, then take the positive root since $y > 0$.

### Part (a)

$y\,dy = 2x\,dx$ **M1**

$\frac{y^2}{2} = x^2 + C$ **A1**

$\frac{3^2}{2} = 1^2 + C \;\Rightarrow\; C = \frac{7}{2}$ **M1**

$y^2 = 2x^2 + 7$ **A1**

$y = \sqrt{2x^2 + 7}$ **A1**

> $y = \sqrt{2x^2 + 7}$.

### Part (b)

$y = \sqrt{2(4)^2 + 7} = \sqrt{39}$ **M1**

$= 6.24\ldots$ **A1**

> When $x = 4$, $y = \sqrt{39} = 6.24$ (3 s.f.).

> **Examiner note** Take the positive square root because the problem states $y > 0$. A single constant $C$ (not two) is correct when integrating both sides simultaneously.

**Total: [7 marks]**

## Question 9

*[7 marks]*

Complex number in modulus–argument form.

### Given

$z = 2 - 2\sqrt{3}\,\mathrm{i}$.

### Formula / Law used

$|z| = \sqrt{a^2 + b^2}$, $\arg z = \arctan\left(\frac{b}{a}\right)$ (correct quadrant); De Moivre's theorem $z^n = |z|^n\left(\cos n\theta + \mathrm{i}\sin n\theta\right)$.

### Strategy

Compute modulus and argument (a); apply De Moivre for $z^6$ (b); determine when $n\theta$ is a multiple of $\pi$ for $z^n$ real (c).

### Thought process

$z$ is in the fourth quadrant ($a > 0$, $b < 0$), so the argument is negative: $-\frac{\pi}{3}$. A power is real when its argument is a whole multiple of $\pi$.

> **Hints**
>
> **Hint 1:** Plot $z$ to decide the quadrant before choosing the sign of the argument.
> **Hint 2:** Use De Moivre's theorem to raise the modulus–argument form to a power.
> **Hint 3:** $z^n$ is real exactly when $n\theta$ is an integer multiple of $\pi$; here $\theta = -\frac{\pi}{3}$.

### Part (a)

$|z| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12}$ **M1**

$|z| = 4$ **A1**

$\arg z = -\dfrac{\pi}{3} \quad (= -60^\circ)$ **A1**

$|z| = 4, \;\; \arg z = -\dfrac{\pi}{3}.$

### Part (b)

$z^6 = 4^6\left(\cos\left(6 \cdot \left(-\frac{\pi}{3}\right)\right) + \mathrm{i}\sin\left(6 \cdot \left(-\frac{\pi}{3}\right)\right)\right) = 4096\left(\cos(-2\pi) + \mathrm{i}\sin(-2\pi)\right)$ **M1**

$z^6 = 4096$ **A1**

$z^6 = 4096$ (i.e. $4096 + 0\,\mathrm{i}$).

### Part (c)

$z^n$ real $\iff n \cdot \left(-\frac{\pi}{3}\right) = k\pi,\; k \in \mathbb{Z} \iff \frac{n}{3} \in \mathbb{Z}$ **M1**

least positive $n = 3$ **A1**

$n = 3$ (indeed $z^3 = -64$, which is real).

> **Examiner note** A power is real when its imaginary part vanishes, i.e. the argument is a multiple of $\pi$, not only of $2\pi$ — so $z^3 = -64$ already counts.

**Total: [7 marks]**

## Question 10

*[16 marks]*

Logistic population model with Euler's method.

### Given

$\dfrac{dP}{dt} = 0.4P\left(1 - \dfrac{P}{25}\right)$, $P(0) = 3$ (thousand fish).

### Formula / Law used

Logistic equation; carrying capacity; Euler's method $P_{n+1} = P_n + h\,f(P_n)$; the explicit logistic solution.

### Strategy

Read $K$ and evaluate the rate at $t = 0$ (a); two Euler steps for (b); fit $A$ from the initial condition and evaluate (c); solve $P = 20$ (d); take the limit (e).

### Thought process

The carrying capacity is the value that makes the bracket zero, i.e. $25$. As $t \to \infty$ the population approaches this ceiling.

> **Hints**
>
> **Hint 1:** The carrying capacity is the population at which $\dfrac{dP}{dt} = 0$ (with $P \neq 0$).
>
> **Hint 2:** Euler's method: $P_{n+1} = P_n + h \times$ (rate at $P_n$); with $h = 0.5$ you need two steps to reach $t = 1$.
>
> **Hint 3:** Find $A$ from $P(0) = 3$, then evaluate the explicit formula; solve $P(t) = 20$ by graphing.

### Part (a)

carrying capacity $= 25$ thousand fish **A1**

$\left.\dfrac{dP}{dt}\right|_{t=0} = 0.4(3)(1 - \frac{3}{25}) = 1.06$ (thousand per year) **A1**

### Part (b)

> **GDC Method**
>
> Set $f(P) = 0.4P(1 - P/25)$. Iterate $P_{n+1} = P_n + 0.5\,f(P_n)$ from $P_0 = 3$.

$P_1 = 3 + 0.5\big(0.4(3)(1 - \frac{3}{25})\big) = 3.528$ **M1**

$P_1 = 3.528$ **A1**

$P_2 = 3.528 + 0.5\big(0.4(3.528)(1 - \frac{3.528}{25})\big)$ **M1**

$P(1) \approx P_2 = 4.13\ldots$ **A1**

> Euler's method gives $P(1) \approx 4.13$ thousand fish.

### Part (c)

$P(0) = \dfrac{25}{1 + A} = 3 \;\Rightarrow\; 1 + A = \dfrac{25}{3}$ **M1**

$A = \dfrac{22}{3} = 7.33\ldots$ **A1**

$P(t) = \dfrac{25}{1 + 7.333\ldots\,\mathrm{e}^{-0.4t}}$ **A1**

$P(6) = \dfrac{25}{1 + 7.333\ldots\,\mathrm{e}^{-0.4(6)}}$ **M1**

$= 15.0\ldots$ **A1**

> $A = \dfrac{22}{3}$ and $P(6) = 15.0$ thousand fish (3 s.f.).

## Part (d)

> **GDC Method**
>
> Graph $y_1 = \dfrac{25}{1 + 7.333\mathrm{e}^{-0.4x}}$ and $y_2 = 20$; use `intersect`.

$\dfrac{25}{1 + 7.333\ldots\,\mathrm{e}^{-0.4t}} = 20$ **M1**

intersection of the curve with $y = 20$ **GDC M1**

$t = 8.45\ldots$ **A1**

> The population reaches 20 thousand after $t = 8.45$ years (3 s.f.).

## Part (e)

As $t \to \infty$, $\mathrm{e}^{-0.4t} \to 0$, so $P(t) \to \dfrac{25}{1} = 25$. **R1**

The population levels off and approaches the carrying capacity of $25$ thousand fish. **A1**

> In the long term $P \to 25$ thousand fish (the carrying capacity).

> **Examiner note** For Euler's method, show each intermediate value; a single final number with no iterations shown earns only the answer mark. Keep $A$ stored exactly as $\frac{22}{3}$.

**Total: [16 marks]**

## Question 11

*[18 marks]*

Calculus on $f(x) = x^2 e^{-x}$: maximum, area, volume, inflection.

### Given

$f(x) = x^2 e^{-x}$, $x \geq 0$; region $R$ under the curve from $x = 0$ to $x = 4$.

### Formula / Law used

Product rule for $f'$; stationary points where $f' = 0$; area $\int f\,dx$; volume $\pi \int f^2\,dx$; inflection where $f'' = 0$.

### Strategy

Differentiate for the maximum (a); numerically integrate for area (b) and volume (c); use the second derivative for inflection (d); take the limit for (e).

### Thought process

The factor $x^2$ pushes the curve up while $e^{-x}$ drags it down; the exponential dominates for large $x$, so $f \to 0$.

> **Hints**
>
> **Hint 1:** Differentiate with the product rule and factorise to find the stationary points.
> **Hint 2:** Volume of revolution about the $x$-axis is $\pi \int_a^b \left(f(x)\right)^2 dx$.
> **Hint 3:** Points of inflection occur where $f''(x) = 0$ and the concavity changes.

### Part (a)

$f'(x) = 2x\,e^{-x} - x^2 e^{-x} = x\,e^{-x}(2 - x)$ **M1**

$f'(x) = 0 \;\Rightarrow\; x = 0$ or $x = 2$ **A1**

$f(2) = 2^2\,e^{-2} = 4e^{-2}$ **M1**

$f(2) = 0.541\ldots$ **A1**

> Local maximum at $(2,\ 0.541)$ (3 s.f.).

### Part (b)

> **GDC Method**
>
> Compute $\displaystyle\int_0^4 x^2 e^{-x}\,dx$ numerically.

Area $= \displaystyle\int_0^4 x^2 e^{-x}\,dx$ **M1**

numerical integration **GDC M1**

$= 1.52\ldots$ **A1**

> Area of $R = 1.52$ (3 s.f.).

### Part (c)

> **GDC Method**
>
> Compute $\pi \displaystyle\int_0^4 \left(x^2 e^{-x}\right)^2 dx = \pi \int_0^4 x^4 e^{-2x}\,dx$.

$V = \pi \int_0^4 \left(x^2 e^{-x}\right)^2 dx$ **M1**

$= \pi \int_0^4 x^4 e^{-2x} dx$ **A1**

numerical integration **GDC M1**

$= 2.12\ldots$ **A1**

> Volume $= 2.12$ (3 s.f.).

### Part (d)

$f''(x) = e^{-x}\left(x^2 - 4x + 2\right)$ **M1**

$x^2 - 4x + 2 = 0$ **M1**

$x = 2 - \sqrt{2} = 0.586$ **A1**

$x = 2 + \sqrt{2} = 3.41$ **A1**

> Points of inflection at $x = 0.586$ and $x = 3.41$ (3 s.f.).

### Part (e)

As $x \to \infty$, the exponential decay $e^{-x}$ dominates the polynomial growth $x^2$. **M1**

Hence $x^2 e^{-x} \to 0$. **R1**

$\lim_{x\to\infty} f(x) = 0 \quad (y = 0$ is a horizontal asymptote) **A1**

> $f(x) \to 0$ as $x \to \infty$; the $x$-axis is a horizontal asymptote.

> **Examiner note** For the volume, remember to square the function *and* include the factor $\pi$. A frequent error is integrating $f$ instead of $f^2$.

**Total: [18 marks]**

## Question 12

*[18 marks]*

Normal model linked to a binomial application.

### Given

Processing time $X \sim N(6.5,\ 1.2^2)$ minutes; samples of 15 orders.

### Formula / Law used

Normal CDF and inverse normal; binomial distribution with $p = P(X > 8)$; conditional probability.

### Strategy

Normal probabilities (a)–(b); inverse normal (c); binomial with the success probability from (a) for (d)–(e),(g); conditional probability (f).

### Thought process

The probability that one order takes more than 8 minutes becomes the "success" probability for the binomial model over 15 orders.

> **Hints**
>
> **Hint 1:** Decide for each part whether you need a probability (CDF) or a value (inverse normal).
> **Hint 2:** The number of long orders out of 15 is binomial with $p = P(X > 8)$ from part (a).
> **Hint 3:** For (f), $\{X > 8\} \subset \{X > 6.5\}$, so divide $P(X > 8)$ by $P(X > 6.5) = 0.5$.

### Part (a)

> **GDC Method**
>
> `normalcdf(8,1E99,6.5,1.2)`.

$P(X > 8)$ **GDC M1**

$= 0.106\ldots$ **A1**

> $P(X > 8) = 0.106$ (3 s.f.).

### Part (b)

$P(5 < X < 7) = \texttt{normalcdf}(5, 7, 6.5, 1.2)$ **GDC M1**

$= 0.556\ldots$ **A1**

> $P(5 < X < 7) = 0.556$ (3 s.f.).

### Part (c)

> **GDC Method**
>
> `invNorm(0.90,6.5,1.2)` (left-tail area $0.90$).

Require $t$ with $P(X < t) = 0.90.$ **M1**

$t = \texttt{invNorm}(0.90,\ 6.5,\ 1.2)$ **GDC M1**

$t = 8.04\ldots$ **A1**

> $90\%$ of orders are processed within $8.04$ minutes (3 s.f.).

### Part (d)

Let $Y$ be the number of long orders out of 15, so $Y \sim B(15, 0.1056\ldots)$. **M1**

> **GDC Method**
>
> `binompdf(15,0.1056,3)`.

$P(Y = 3) = \binom{15}{3}(0.1056)^3(0.8944)^{12}$ **M1**

$= 0.141\ldots$ **A1**

$P(Y = 3) = 0.141$ (3 s.f.).

### Part (e)

$P(Y \geq 2) = 1 - P(Y \leq 1)$ **M1**

$= 0.481\ldots$ **A1**

$P(Y \geq 2) = 0.481$ (3 s.f.).

### Part (f)

$P(X > 8 \mid X > 6.5) = \dfrac{P(X > 8)}{P(X > 6.5)}$ **M1**

$P(X > 6.5) = 0.5$ (mean of a symmetric distribution) **A1**

$= \dfrac{0.1056\ldots}{0.5}$ **M1**

$= 0.211\ldots$ **A1**

$P(X > 8 \mid X > 6.5) = 0.211$ (3 s.f.).

### Part (g)

$E(Y) = np = 15 \times 0.1056\ldots$ **M1**

$= 1.58\ldots$ **A1**

Expected number of long orders is $1.58$.

> **Examiner note** $P(X > 6.5) = 0.5$ exactly, since $6.5$ is the mean of a symmetric distribution — a useful shortcut in part (f). Use the unrounded $p = 0.1056$ for the binomial parts.

**Total: [18 marks]**

End of worked solutions — Mock Exam 2