IB Demystified

Examiners · Moderators · Mentors

Mathematics: Analysis and Approaches

Higher Level

Paper 1 — Mock Examination

Mock Exam 3

Non-calculator

Time allowed: **2 hours**

Maximum mark: **110 marks**

Candidate information

Candidate name: ....

Session number: ....

Date: ....

Topic emphasis: integration, differential equations, vectors, trigonometric identities and algebraic reasoning

Instructions to candidates

- Write your session number in the box on the cover.
- Do not begin this paper until you are told to do so.
- The use of any calculator is **not** permitted for this paper.
- **Section A:** answer every question. Record your answers in the boxes that follow each question; you may continue below the printed lines if you need more room.
- **Section B:** answer every question. Set out your solutions on separate answer-booklet pages, starting each question on a fresh page.
- Unless a question states otherwise, give numerical answers exactly, or rounded to three significant figures.
- Where a result is requested exactly, decimal approximations will not earn full marks.
- Show all of your reasoning. Marks may not be awarded for an answer that is not supported by working.
- A clean copy of the *Mathematics: Analysis and Approaches HL formula booklet* may be used.
- The maximum mark for this paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no supporting work. Where an answer is incorrect, marks may still be given for a valid method, provided the method is clearly shown. You are therefore strongly advised to show all working.

Section A

*Answer **all** questions. Write your answers in the boxes provided. Working may be continued below the lines if necessary.*

1. **[Maximum mark: 5]**

(a) Using the substitution $u = x^2 + 1$, or otherwise, find $\int x\sqrt{x^2+1}\,\mathrm{d}x$. **[3]**

(b) Hence evaluate $\int_0^{\sqrt{3}} x\sqrt{x^2+1}\,\mathrm{d}x$. **[2]**

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2. **[Maximum mark: 6]**

Prove the identity $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} \equiv \dfrac{2}{\sin\theta}$.

3. **[Maximum mark: 6]**

The vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$. **[1]**

(b) Find the angle between **a** and **b**. **[3]**

(c) Find a unit vector that is perpendicular to both **a** and **b**. **[2]**

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4. **[Maximum mark: 7]**

Consider the differential equation $\frac{dy}{dx} = xy$, where $y > 0$.

(a) Find the general solution of the differential equation. **[4]**

(b) Given that $y = 2$ when $x = 0$, find the particular solution, giving y in terms of x. **[3]**

5. **[Maximum mark: 7]**

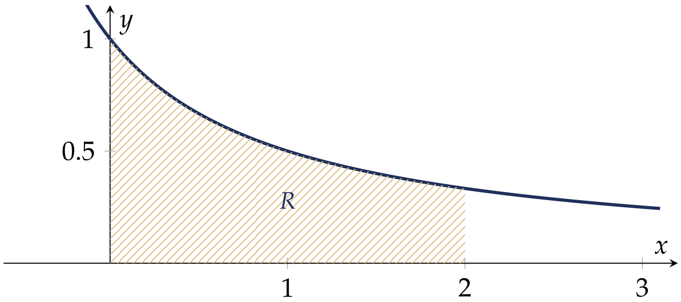
The polynomial $p(x) = 2x^3 + ax^2 - 5x + b$, where $a, b \in \mathbb{R}$, leaves a remainder of 12 when divided by $(x - 2)$. Also, $(x + 1)$ is a factor of $p(x)$.

(a) Show that $4a + b = 6$ and $a + b = -3$. **[4]**

(b) Hence find the values of a and b. **[3]**

6. **[Maximum mark: 8]**

The region R is enclosed by the curve $y = \dfrac{1}{x+1}$, the x-axis, and the lines $x = 0$ and $x = 2$, as shown below.



(a) Find the area of R. **[3]**

(b) The region R is rotated through 360° about the x-axis. Find the exact volume of the solid of revolution formed. **[5]**

7. **[Maximum mark: 9]**

(a) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **[4]**

(b) Hence solve $\sin 3\theta + \sin\theta = 0$ for $0 \le \theta \le \pi$. **[5]**

8. **[Maximum mark: 10]**

(a) Show that $\sin^2 x = \dfrac{1 - \cos 2x}{2}$. **[1]**

(b) Hence find $\int \sin^2 x \, dx$. **[4]**

(c) The region bounded by the curve $y = \sin x$, the x-axis and the lines $x = 0$ and $x = \pi$ is rotated through $360°$ about the x-axis. Find the exact volume of the solid generated. **[5]**

Section B

*Answer **all** questions in the answer booklet provided. Please begin each question on a new page.*

9. **[Maximum mark: 12]**

A population of bacteria, N, grows at a rate proportional to its size. At time $t = 0$ hours there are 50 bacteria, and after 2 hours there are 200 bacteria.

(a) Write down a differential equation for $\frac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N and a positive constant k. **[1]**

(b) Solve this differential equation to show that $N = 50\mathrm{e}^{kt}$. **[4]**

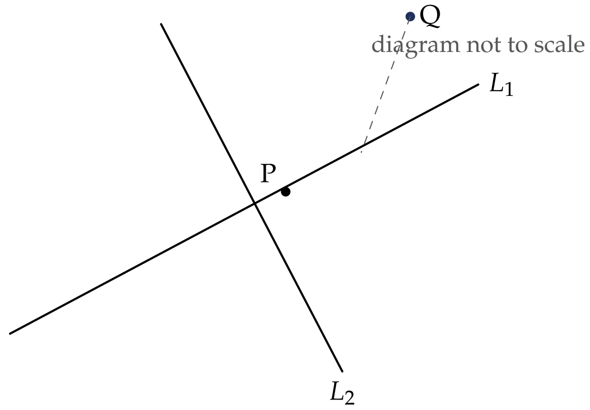
(c) Find the exact value of k. **[3]**

(d) Find the time, in hours, at which the population first reaches 800. **[4]**

10. **[Maximum mark: 13]**

Two lines are defined by

$$L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \qquad L_2 : \mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$



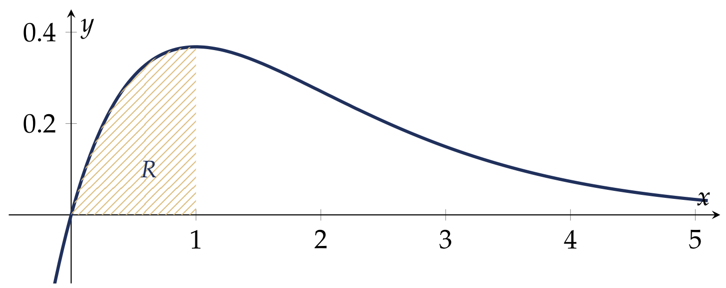
(a) Show that L_1 and L_2 intersect, and find the coordinates of the point of intersection P. **[6]**

(b) Find the acute angle between L_1 and L_2. **[4]**

(c) The point Q has position vector $\begin{pmatrix} 7 \\ 0 \\ 5 \end{pmatrix}$. Find the shortest distance from Q to the line L_1. **[3]**

11. **[Maximum mark: 13]**

Consider the function $f(x) = x\mathrm{e}^{-x}$. Part of the graph of $y = f(x)$ is shown below.



(a) Find $\int x\mathrm{e}^{-x}\,\mathrm{d}x$. **[4]**

(b) The region R is bounded by the curve, the x-axis and the lines $x = 0$ and $x = 1$. Find the exact area of R. **[4]**

(c) Find the coordinates of the maximum point of $y = f(x)$. **[3]**

(d) Hence find the exact value of $\int_0^2 x\mathrm{e}^{-x}\,\mathrm{d}x$. **[2]**

12. **[Maximum mark: 14]**

A population P, measured in thousands, at time t years satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{50}P(10 - P), \qquad 0 \leq P < 10,$$

with $P = 2$ when $t = 0$.

(a) Show that $\frac{1}{P(10-P)} = \frac{1}{10}\left(\frac{1}{P} + \frac{1}{10-P}\right)$. **[3]**

(b) Hence show that the general solution may be written as $\ln\left(\frac{P}{10-P}\right) = \frac{t}{5} + c$, where c is a constant. **[5]**

(c) Given that $P = 2$ when $t = 0$, find the value of c. **[2]**

(d) Find the value of P when $t = 5\ln 4$. **[4]**

End of examination.