IB Demystified

Mathematics: Analysis and Approaches Higher Level

Paper 2 — Mock Examination

Mock Exam 3

Graphic display calculator required

Time allowed: 2 hours

Maximum mark: 110

Information for candidates

- Do not begin this paper until you are told that you may do so.
- A graphic display calculator is needed throughout this examination.
- You may use a clean copy of the *Mathematics: Analysis and Approaches HL formula booklet*.
- Answer every question.
- Section A answers must be written in the working space provided beneath each question.
- Section B answers must be written on the continuation pages, beginning each question on a fresh page.
- Give numerical answers exactly, or rounded to three significant figures, unless a question instructs otherwise.
- Any answer obtained from a calculator must be backed up by suitable written working; full marks are not guaranteed for an unsupported result.
- When a graph is used to solve a problem, include a sketch or a clear description of the graphing approach.
- When you rely on regression, equation solving, numerical integration, a normal or binomial model, or a graph intersection, the method must be shown clearly.
- The maximum mark for this paper is **110**. The time allowed is **2 hours**.

Candidate information

Candidate name: ______________________________

Session number: ______________________________

Date: ______________________________

Section A

Answer all questions in the spaces provided · 58 marks

Full marks may not be awarded for answers given without supporting working. Where a calculator is used to obtain a result, indicate the method clearly.

1. *[Maximum mark: 7]*

The second term of a geometric series is 12 and its fifth term is 1.5.

(a) Find the common ratio and the first term of the series. [3]

(b) Find the sum to infinity of the series. [2]

(c) Find the least number of terms required for the partial sum to be within 0.01 of the sum to infinity. [2]

2.

[Maximum mark: 6]

Consider the function $f(x) = \dfrac{2x-6}{(x-2)(x+1)}$.

(a) Write down the equations of all the asymptotes of the graph of f. [3]

(b) Find the coordinates of the points where the graph of f meets the coordinate axes. [3]

3. *[Maximum mark: 5]*

A continuous random variable X has probability density function

$$f(x) = \begin{cases} k(4 - x^2), & 0 \le x \le 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Find the value of k. [2]

(b) Find the expected value $E(X)$. [3]

4. *[Maximum mark: 6]*

The temperature T, in °C, inside a greenhouse is modelled by

$$T(t) = 18 + 6\cos\left(\frac{\pi}{12}(t-15)\right), \qquad 0 \le t \le 24,$$

where t is the time in hours after midnight.

(a) Write down the maximum temperature, and the time at which it occurs. [3]

(b) Find the times during the day at which the temperature is exactly $20\,°$C. [3]

5. *[Maximum mark: 7]*

A line L passes through the points $A(3,-1,2)$ and $B(5,2,4)$. A plane Π has equation $x+2y-z=4$.

(a) Write down a vector equation of the line L. [2]

(b) Find the coordinates of the point at which L meets the plane Π. [3]

(c) Find the acute angle between L and the plane Π. [2]

6. *[Maximum mark: 6]*

A screening test for a medical condition is used in a population in which 4% of people have the condition. If a person has the condition, the test is positive with probability 0.95. If a person does not have the condition, the test is negative with probability 0.92.

(a) Find the probability that a randomly chosen person tests positive. [3]

(b) Given that a person tests positive, find the probability that they actually have the condition. [3]

7. *[Maximum mark: 7]*

A closed cylindrical can has radius r cm, height h cm and a fixed volume of 330 cm^3.

(a) Show that the total surface area S, in cm^2, is given by

$$S = 2\pi r^2 + \frac{660}{r}.$$

[2]

(b) Find the value of r that minimises the surface area. [3]

(c) Hence find the minimum surface area. [2]

8. *[Maximum mark: 7]*

(a) On the same axes, sketch the graphs of $y = 2^x$ and $y = x^2$, and find the x-coordinates of their points of intersection. [4]

(b) Hence solve the inequality $2^x \leq x^2$. [3]

9. *[Maximum mark: 7]*

(a) Find the coefficient of x^3 in the expansion of $(2 + 3x)^7$. [3]

(b) Find the constant term in the expansion of $\left(x^2 - \dfrac{1}{x}\right)^9$. [4]

Section B

Answer all questions on the continuation pages · 52 marks

Begin each question on a new page of the answer booklet.

10. *[Maximum mark: 16]*

Consider the function $f(x) = \dfrac{x^2+3}{x-1}$, for $x \neq 1$.

(a) Find the equation of the vertical asymptote, and show that the line $y = x + 1$ is an oblique asymptote of the graph of f. [4]

(b) Show that the graph of f has no x-intercepts, and state the y-intercept. [2]

(c) Find the coordinates of the local maximum point and the local minimum point of f. [5]

(d) Hence write down the range of f. [3]

(e) Find the value of x for which $f(x) = x + 2$. [2]

11. *[Maximum mark: 18]*

A Ferris wheel has radius 22 m and its centre is 25 m above the ground. A passenger boards at the lowest point of the wheel at time $t = 0$ seconds. The wheel turns at a constant rate, completing one full revolution every 40 seconds. The height h metres of the passenger above the ground is modelled by

$$h(t) = 25 - 22\cos\left(\frac{\pi t}{20}\right).$$

(a) Write down the maximum and minimum heights of the passenger, and the period of the motion. [3]

(b) Find the height of the passenger when $t = 10$. [2]

(c) Find the first two times at which the passenger is at a height of 35 m. [3]

(d) Find the rate at which the height is changing when $t = 15$. [3]

(e) Find the length of time, during one revolution, for which the passenger is more than 40 m above the ground. [4]

(f) Find the maximum upward speed of the passenger and the first time at which it occurs. [3]

12. *[Maximum mark: 18]*

The plane Π has equation $2x + y - 2z = 6$. The points $A(2, 4, 1)$ and $B(5, 2, -1)$ are given.

(a) Verify that the point A lies on the plane Π. [2]

(b) Find the shortest distance from B to the plane Π. [4]

(c) Find the coordinates of the foot of the perpendicular from B to the plane Π. [4]

(d) Find the coordinates of B', the reflection of B in the plane Π. [3]

(e) Find the acute angle between the line AB and the plane Π. [5]

End of examination