IB Demystified

EXAMINERS MODERATORS MENTORS

Mathematics: Analysis and Approaches
Standard Level

Paper 1 – Mock Examination

Mock Exam 3

Question Paper

Non-calculator

Time allowed: 1 hour 30 minutes

Maximum mark: **80 marks**

Mathematics: Analysis and Approaches SL – Paper 1 (Non-calculator)

Instructions to Candidates

- Do not open this examination paper until you are told to do so.
- **A calculator is not permitted** for this paper.
- A clean copy of the *Mathematics: Analysis and Approaches SL formula booklet* may be used.
- Answer **all** questions.
- **Section A:** write your answers in the answer boxes provided. Working may be continued below the lines if required.
- **Section B:** write your answers in the answer booklet or on the continuation pages provided. **Start each question on a new page.**
- Unless a question states otherwise, give numerical answers exactly or correct to three significant figures.
- Exact answers are preferred wherever possible.
- Show all working. Full marks may not be awarded for a correct answer that is not supported by working.
- The maximum mark for this paper is **80 marks**.
- The time allowed is **1 hour 30 minutes**.

Candidate name: . . . .

Session number: . . . .

Date: . . . .

A clean copy of the formula booklet is required for this paper.

Section A

Full marks are not necessarily awarded for a correct answer with no working. Where an answer is incorrect, some marks may be awarded for correct method, provided this is shown by written working. You are advised to show all working.

Answer all questions in the answer boxes provided.

1. **[Maximum mark: 5]**

The sum of the first n terms of a sequence is given by $S_n = 2n^2 + 3n$, for $n \in \mathbb{Z}^+$.

(a) Find the first term of the sequence. [1]

(b) Show that the nth term of the sequence is $u_n = 4n + 1$. [3]

(c) Hence state the common difference of the sequence. [1]

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2. **[Maximum mark: 5]**

Consider the quadratic expression $2x^2+3x-5$.

(a) Factorise $2x^2+3x-5$ completely. [2]

(b) Hence solve the inequality $2x^2+3x\leq 5$. [3]

3. **[Maximum mark: 7]**

Consider the infinite geometric series

$$\sum_{k=1}^{\infty} 6\left(\frac{1}{3}\right)^{k-1}.$$

(a) Write down the first term and the common ratio of the series. [2]

(b) Find the sum to infinity of the series. [2]

(c) Find the exact sum of the first four terms of the series, giving your answer as a fraction in lowest terms. [3]

4. **[Maximum mark: 4]**

The point $P(2,5)$ lies on the graph of $y = f(x)$. A new function is defined by

$$g(x) = f(x+1) - 3.$$

(a) Describe fully the two transformations that map the graph of $y = f(x)$ onto the graph of $y = g(x)$. [2]

(b) Find the coordinates of the image of P on the graph of $y = g(x)$. [2]

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5. **[Maximum mark: 6]**

Consider the curve with equation $y = x^2 - 4x + 1$.

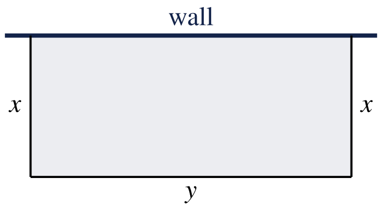
(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the tangent to the curve at the point where $x = 3$, giving your answer in the form $y = mx + c$. [2]

(c) Find the coordinates of the point on the curve at which the tangent is horizontal. [2]

6. **[Maximum mark: 8]**

A rectangular enclosure is to be formed against a long straight wall. The wall forms one complete side of the enclosure, and a total of 60 metres of fencing is used for the remaining three sides. The two sides perpendicular to the wall each have length x metres.



(a) Show that the area A enclosed, in square metres, is given by $A = 60x - 2x^2$. [2]

(b) Find the value of x that gives the maximum area, justifying that it is a maximum. [4]

(c) Find the maximum area of the enclosure. [2]

Section B

Answer all questions in the answer booklet or on the continuation pages provided. Start each question on a new page.

Full marks are not necessarily awarded for a correct answer with no working. Where an answer is incorrect, some marks may be awarded for correct method, provided this is shown by written working. You are advised to show all working.

7. **[Maximum mark: 13]**

Two saving plans each run for a number of months.

Under **Plan A**, the amount saved in successive months forms an arithmetic sequence. The amount saved in the first month is \$20, and the amount saved increases by \$6 each month.

Under **Plan B**, the amount saved in successive months forms a geometric sequence. The amount saved in the first month is \$2, and the amount saved doubles each month.

(a) For Plan A, find the amount saved in the 12th month. [3]

(b) For Plan A, find the total amount saved over the first 12 months. [2]

(c) For Plan B, find the total amount saved over the first 10 months. [3]

(d) Find the first month in which the amount saved under Plan B exceeds the amount saved under Plan A in that same month. [5]

8. **[Maximum mark: 15]**

The function f is defined by $f(x) = x^3 - 6x^2 + 9x$, for $x \in \mathbb{R}$.

(a) Show that $f(x) = x(x-3)^2$. [2]

(b) Hence write down the coordinates of the points where the graph of $y = f(x)$ meets the x-axis. [2]

(c) Find the coordinates of the stationary points of the graph of $y = f(x)$, and determine the nature of each. [5]

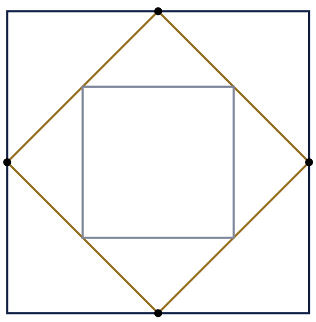
(d) Sketch the graph of $y = f(x)$, showing clearly the intercepts and the stationary points. [2]

(e) Find the exact area enclosed between the graph of $y = f(x)$ and the x-axis from $x = 0$ to $x = 3$. [4]

9. **[Maximum mark: 17]**

A sequence of squares is constructed as follows. The first square, C_1, has side length 8 cm. Each subsequent square is formed by joining the midpoints of the sides of the previous square, as shown in the diagram.

diagram not to scale



C_1 (side 8 cm)

The side lengths of the squares $C_1, C_2, C_3, \ldots$ form a geometric sequence.

(a) Show that the side length of the second square C_2 is $4\sqrt{2}$ cm. [3]

(b) Write down the common ratio of the sequence of side lengths. [2]

(c) Let A_n denote the area of square C_n. Show that the areas $A_1, A_2, A_3, \ldots$ form a geometric sequence with common ratio $\frac{1}{2}$. [2]

(d) Find the total area of the first six squares, giving your answer in cm^2. [4]

(e) Find the total area of all the squares in the infinite sequence. [2]

(f) Find the total perimeter of all the squares in the infinite sequence, giving your answer in the form $(p + q\sqrt{2})$ cm, where $p, q \in \mathbb{Z}^+$. [4]

Section B — Answer Booklet / Continuation Pages

Start each question on a new page. Write the question number clearly.

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