IB Demystified

Mathematics: Analysis and Approaches SL

Paper 2 Mock Examination — Question Paper

Mock Exam 1

Graphic Display Calculator Required

Time allowed: 1 hour 30 minutes

Maximum mark: **80 marks**

Information for Candidates

Candidate Information

Candidate name: . . . .

Session number: . . . .

Date: . . . .

Read the following before you begin.

- Do not open this examination paper until you are told to do so.
- A graphic display calculator (GDC) is required throughout this paper.
- A clean copy of the **Mathematics: Analysis and Approaches SL formula booklet** may be used.
- Answer **every** question.
- **Section A:** write each answer inside the answer box provided beneath the question.
- **Section B:** write your answers in the answer booklet or on the continuation pages provided. Begin each Section B question on a fresh page.
- Unless a question states otherwise, give numerical answers *exactly* or correct to **three significant figures**.
- Any answer obtained from a calculator must be supported by suitable working, a sketch, a clear setup or an explanation.
- Where a graph is used to solve a problem, include a sketch or describe the graphing method clearly.
- Unsupported answers may not receive full marks, so you are advised to show all working.
- The maximum mark for this paper is **80 marks**. The time allowed is **1 hour 30 minutes**.

Section A

Answer all questions in the answer boxes provided.

Full marks may not be awarded for an answer given without working. Answers obtained from a graphic display calculator should be supported by suitable working; for example, where a graph is used, include a sketch as part of your answer.

1. **[Maximum mark: 5]**

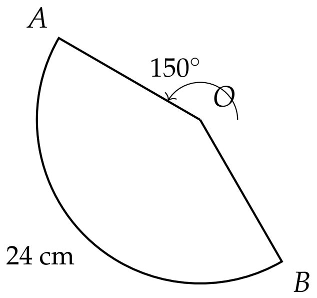
Consider the function $f(x) = \dfrac{(2x+3)^2}{x}$, where $x \in \mathbb{R}$, $x \neq 0$.

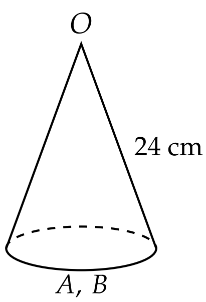
(a) Show that $\dfrac{(2x+3)^2}{x} = 4x + 12 + \dfrac{9}{x}$. **[2]**

(b) Hence find $\displaystyle\int f(x)\,\mathrm{d}x$. **[3]**

2. **[Maximum mark: 6]**

A sheet of card is cut into the shape of a circular sector OAB with centre O, radius 24 cm and angle $A\hat{O}B = 150°$. The straight edges OA and OB are then joined so that the sector forms the curved surface of a hollow cone (with no base). The diagrams below are not to scale.





Find

(a) the area of the sector OAB; **[3]**

(b) the radius of the base of the cone. **[3]**

3. **[Maximum mark: 4]**

The operating time, T hours, of a certain brand of rechargeable lamp is modelled by a normal distribution with mean 42 hours and standard deviation 5 hours.

(a) Find the probability that a randomly chosen lamp operates for more than 50 hours. **[2]**

(b) The manufacturer states that the longest-lasting 15% of lamps each operate for more than t hours. Find the value of t. **[2]**

4. **[Maximum mark: 6]**

On a particular day the temperature, θ °C, inside a greenhouse is modelled by

$$\theta(t) = 18 + 7\sin\left(\frac{\pi t}{12}\right), \qquad 0 \le t \le 24,$$

where t is the number of hours after midnight.

(a) Find the maximum temperature predicted by the model, and the time at which it first occurs. **[3]**

(b) Find the values of t in the interval $0 \le t \le 24$ for which $\theta(t) = 22$. **[3]**

5. **[Maximum mark: 8]**

A fitness study records the number of hours, x, that each of eight participants exercised in a week, together with their score, y, on a wellbeing questionnaire. The results are shown below.

x (hours)	2	3	5	6	8	9	11	12
y (score)	38	45	52	60	68	71	82	88

(a) Find Pearson's product–moment correlation coefficient, r. **[2]**

(b) Write down the equation of the regression line of y on x. **[2]**

(c) Use your regression line to estimate the wellbeing score of a participant who exercised for 7 hours. **[2]**

(d) The study leader wishes to estimate the score of a participant who exercises for 20 hours in a week. Comment on the reliability of using this model for that estimate. **[2]**

6. **[Maximum mark: 6]**

Consider the function $f(x) = x\,\mathrm{e}^{-0.3x}$ for $0 \leq x \leq 8$.

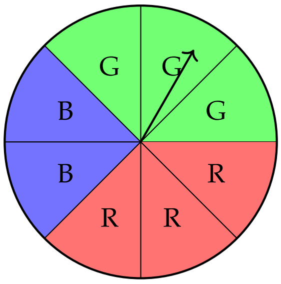
(a) Find the x-coordinate of the maximum point of the graph of $y = f(x)$. **[3]**

(b) Find the area of the region enclosed by the graph of $y = f(x)$, the x-axis and the line $x = 8$. **[3]**

Section B *Answer all questions in the answer booklet or on the continuation pages. Start each question on a new page.*

7. **[Maximum mark: 14]**

At a charity event, players play a game using a fair spinner divided into eight equal sectors. Three sectors are coloured *green*, two are coloured *blue* and three are coloured *red*, as shown.



In one spin, the score obtained is:

- 0 points if the spinner lands on green;
- 5 points if the spinner lands on blue;
- 1 point if the spinner lands on red.

Let the random variable X denote the score obtained in one spin.

(a) Copy and complete the following probability distribution table, and hence find $\mathrm{E}(X)$. **[4]**

x	0	1	5
$\mathrm{P}(X = x)$			

(b) It costs 2 points to play one game. Find the expected net score per game, and state whether the game is favourable to the player. **[2]**

(c) A player spins the spinner 6 times. Find the probability that the spinner lands on blue exactly 2 times. **[3]**

(d) Find the probability that, in these 6 spins, the spinner lands on blue at least 2 times. **[3]**

(e) Find the probability that the spinner lands on blue for the first time on the 4th spin. **[2]**

8. **[Maximum mark: 16]**

Water flows into a reservoir. For $0 \leq t \leq 25$, the rate of flow is modelled by

$$r(t) = 10 + 8\sin(0.4t) - 0.3t \quad \text{litres per minute,}$$

where t is the time in minutes after the flow begins.

(a) Write down the rate of flow at $t = 0$. **[1]**

(b) Find the maximum rate of flow during the interval $0 \leq t \leq 25$, and the time at which it occurs. **[4]**

(c) Find the total volume of water that flows into the reservoir during the first 25 minutes. **[4]**

(d) Find the first time at which the rate of flow falls to 0 litres per minute. **[3]**

(e) The reservoir already holds 50 litres of water when the flow begins. Find the volume of water in the reservoir at $t = 10$ minutes. **[4]**

9. **[Maximum mark: 15]**

Consider the function $f(x) = \dfrac{2x+1}{x-3}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of (i) the vertical asymptote; (ii) the horizontal asymptote of the graph of $y = f(x)$. **[2]**

(b) Find the coordinates of the points where the graph of $y = f(x)$ crosses the x-axis and the y-axis. **[2]**

(c) Find an expression for $f^{-1}(x)$. **[4]**

(d) State the domain and range of f. **[2]**

(e) On the same set of axes, sketch the graph of $y = f(x)$, clearly labelling both asymptotes with their equations and showing the coordinates of the axis intercepts. **[5]**

Answer booklet — continuation page

Question 7

Answer booklet — continuation page

Question 8

Answer booklet — continuation page

Question 9