IB Demystified

Mathematics: Analysis and Approaches SL

Paper 2 Mock Examination — Question Paper

Mock Exam 2

Graphic Display Calculator Required

Time allowed: 1 hour 30 minutes

Maximum mark: **80 marks**

Original practice material. Not affiliated with or endorsed by the International Baccalaureate Organization.

Information for Candidates

Candidate Information

Candidate name: .....

Session number: .....

Date: .....

Read the following before you begin.

- Do not open this examination paper until you are told to do so.
- A graphic display calculator (GDC) is required throughout this paper.
- A clean copy of the **Mathematics: Analysis and Approaches SL formula booklet** may be used.
- Answer **every** question.
- **Section A:** write each answer inside the answer box provided beneath the question.
- **Section B:** write your answers in the answer booklet or on the continuation pages provided. Begin each Section B question on a fresh page.
- Unless a question states otherwise, give numerical answers *exactly* or correct to **three significant figures**.
- Any answer obtained from a calculator must be supported by suitable working, a sketch, a clear setup or an explanation.
- Where a graph is used to solve a problem, include a sketch or describe the graphing method clearly.
- Unsupported answers may not receive full marks, so you are advised to show all working.
- The maximum mark for this paper is **80 marks**. The time allowed is **1 hour 30 minutes**.

Section A

Answer all questions in the answer boxes provided.

Full marks may not be awarded for an answer given without working. Answers obtained from a graphic display calculator should be supported by suitable working; for example, where a graph is used, include a sketch as part of your answer.

1. **[Maximum mark: 5]**

The first term of a geometric sequence is $u_1 = 8$ and the fourth term is $u_4 = 216$. All terms are positive.

(a) Find the common ratio r. **[2]**

(b) Find the sum of the first ten terms of the sequence. **[3]**

2. **[Maximum mark: 6]**

In a large factory, 25% of the components produced by a machine are classed as premium grade. A quality inspector selects a random sample of 12 components. Let X denote the number of premium-grade components in the sample.

(a) Find $\mathrm{P}(X = 4)$. **[2]**

(b) Find $\mathrm{P}(X \geq 4)$. **[2]**

(c) Find the expected number of premium-grade components in the sample. **[2]**

3. **[Maximum mark: 4]**

Consider the functions $f(x) = \ln(x+2)$, for $x > -2$, and $g(x) = e^x - 3$, for $x \in \mathbb{R}$.

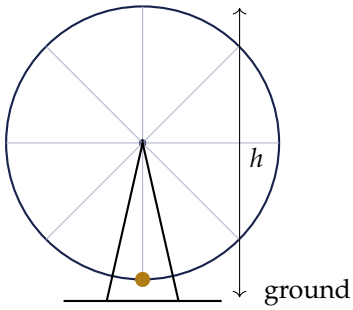
(a) Show that $(g \circ f)(x) = x - 1$. **[2]**

(b) State the domain of $g \circ f$. **[2]**

4. **[Maximum mark: 6]**

A passenger cabin on a Ferris wheel moves so that its height above the ground, h metres, after t seconds is modelled by

$$h(t) = 32 - 28\cos\left(\frac{\pi t}{60}\right), \qquad t \geq 0.$$



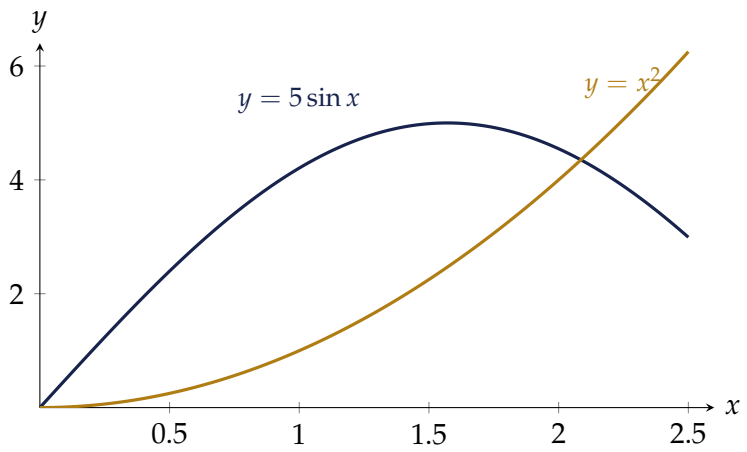
(a) Write down the height of the cabin above the ground at $t = 0$. **[1]**

(b) Find the maximum height of the cabin, and the first time at which it occurs. **[2]**

(c) Find the first time at which the cabin is exactly 50 metres above the ground. **[3]**

5. **[Maximum mark: 8]**

Consider the functions $f(x) = 5\sin x$ and $g(x) = x^2$ for $0 \leq x \leq 2.5$, where x is measured in radians. The graphs of $y = f(x)$ and $y = g(x)$ are shown below.



(a) Use your graph to state the number of solutions of the equation $5\sin x = x^2$ in the interval $0 \leq x \leq 2.5$. **[2]**

(b) Find the non-zero value of x for which $f(x) = g(x)$. **[3]**

(c) Find the area of the region enclosed by the two graphs. **[3]**

6. **[Maximum mark: 6]**

The seats in a concert hall are arranged so that the first row has 22 seats, and each row after the first has 3 more seats than the row in front of it. There are 25 rows in total.

(a) Find the number of seats in the 25th row. **[2]**

(b) Find the total number of seats in the hall. **[2]**

(c) Find the number of the first row that contains more than 60 seats. **[2]**

Section B *Answer all questions in the answer booklet or on the continuation pages. Start each question on a new page.*

7. **[Maximum mark: 14]**

A particle moves in a straight line. Its velocity, v m s^{-1}, at time t seconds is given by

$$v(t) = t^3 - 6t^2 + 8t, \qquad 0 \leq t \leq 5.$$

(a) Find the times at which the particle is instantaneously at rest. **[3]**

(b) Find the acceleration of the particle when $t = 3$. **[2]**

(c) Find the displacement of the particle from its starting position when $t = 5$. **[3]**

(d) Find the total distance travelled by the particle in the interval $0 \leq t \leq 5$. **[4]**

(e) Find the maximum speed of the particle in the interval $0 \leq t \leq 2$. **[2]**

8. **[Maximum mark: 16]**

Consider the function $f(x) = 12x\,\mathrm{e}^{-0.5x}$ for $x \geq 0$.

(a) Write down the value of $f(0)$. **[1]**

(b) Find the coordinates of the maximum point of the graph of $y = f(x)$. **[4]**

(c) Solve the equation $f(x) = 5$. **[4]**

(d) Find the area of the region enclosed by the graph of $y = f(x)$, the x-axis and the line $x = 10$. **[4]**

(e) Describe the behaviour of $f(x)$ as $x \to \infty$, and explain what this represents about the graph. **[3]**

9. **[Maximum mark: 15]**

Two athletes begin a training programme in the same week (week 1).

Athlete A runs 12 km in week 1, and in each following week runs 8% further than in the previous week. The distance Athlete A runs forms a geometric sequence.

Athlete B runs 16 km in week 1, and in each following week runs 0.8 km further than in the previous week. The distance Athlete B runs forms an arithmetic sequence.

(a) Find the distance Athlete A runs in week 6. **[3]**

(b) Find the total distance Athlete A runs during the first 10 weeks. **[3]**

(c) Find the number of the first week in which Athlete A runs more than 25 km. **[3]**

(d) Find the number of the first week in which Athlete A runs further than Athlete B in that week. **[4]**

(e) Explain what happens to the difference between the two athletes' weekly distances in the long term. **[2]**

Answer booklet — continuation page

Question 7

Answer booklet — continuation page

Question 8

Answer booklet — continuation page

Question 9