IB Demystified

Mathematics: Applications and Interpretation HL

Paper 1 | Mock Examination

Mock Exam 3

Calculator: A graphic display calculator (GDC) is required
Time allowed: 2 hours
Maximum mark: 110 marks

Candidate name: ....

Session number: ....

Date: ....

Information for Candidates

- A graphic display calculator is required throughout this paper.
- You may use one clean copy of the *Mathematics: Applications and Interpretation HL* formula booklet.
- Attempt every question. Write each response in the working space provided for that question.
- Unless a question states otherwise, give numerical answers either exactly or rounded to three significant figures.
- Where a question concerns an amount of money, give your answer to two decimal places unless told otherwise.
- Any answer obtained from a calculator must be accompanied by appropriate supporting working.
- If you use a graph to reach an answer, include a sketch or a clear description of the graphing method you used.
- Answers stated without supporting working may not receive full marks.
- The maximum mark for this paper is **110 marks**. The time allowed is **2 hours**.

Full marks are not necessarily awarded for a correct answer with no working. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Question 1 **[Maximum mark: 6]**

onsider the complex numbers $z_1 = 3 + 4i$ and $z_2 = 1 - 2i$.

(a) Find $z_1 z_2$, giving your answer in the form $a + bi$. [2]

(b) Find the modulus and the argument of z_1. [2]

(c) Find $\frac{z_1}{z_2}$, giving your answer in the form $a + bi$. [2]

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Question 2 **[Maximum mark: 7]**

complex number is given by $z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$.

(a) Write z in the form $a + bi$, giving exact values. [2]

(b) Find z^3, giving your answer in both modulus–argument form and in the form $a + bi$. [3]

(c) Write z in Euler form $re^{i\theta}$. [2]

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Question 3 **[Maximum mark: 8]**

ach weekday a commuter travels by either car (C) or train (T). If she travels by car one day, the probability she travels by car the next day is 0.7. If she travels by train one day, the probability she travels by car the next day is 0.4. This is modelled by the transition matrix

$$M = \begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix}, \qquad \text{acting on the state vector } \begin{pmatrix} C \\ T \end{pmatrix}.$$

(a) On Monday she travels by car. Find the probability she travels by car on Wednesday. [2]

(b) Find the long-term proportion of days on which she travels by car and by train. [4]

(c) Interpret your answer to part (b) in context. [2]

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Question 4 **[Maximum mark: 9]**

n open-topped storage box has a square base of side x cm and height h cm. The box must hold a volume of 4000 cm^3.

(a) Show that the external surface area, S cm^2, of the box (base and four sides) is given by

$$S = x^2 + \frac{16000}{x}.$$

[3]

(b) Find the value of x that minimises the surface area. [3]

(c) Find the minimum surface area and the corresponding height of the box. [3]

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■ Question 5 **[Maximum mark: 7]**

quantity y satisfies the differential equation

$$\frac{dy}{dx} = 0.5x + y, \qquad y = 1 \text{ when } x = 0.$$

(a) Use Euler's method with a step length of $h = 0.5$ to estimate the value of y when $x = 1$. [2]

(b) Use Euler's method with a step length of $h = 0.25$ to estimate the value of y when $x = 1$. [3]

(c) State, with a reason, which of your two estimates is likely to be more accurate. [2]

Question 6 **[Maximum mark: 8]**

call centre receives calls at an average rate of 3 calls per 10-minute interval. The number of calls in any 10-minute interval is modelled by a Poisson distribution.

(a) Find the probability of receiving exactly 2 calls in a 10-minute interval. [1]

(b) Find the probability of receiving at least 4 calls in a 10-minute interval. [2]

(c) Find the probability of receiving a total of exactly 15 calls over five consecutive 10-minute intervals. [2]

(d) In a period of six 10-minute intervals, find the probability that exactly 4 of these intervals each receive at least 4 calls. [3]

Question 7 **[Maximum mark: 7]**

spherical weather balloon is being inflated. Air is pumped in at a constant rate of 50 cm^3 per second. At a certain instant the radius of the balloon is 8 cm. The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$ and its surface area is $A = 4\pi r^2$.

(a) Find the rate at which the radius is increasing at this instant. [3]

(b) Find the rate at which the surface area is increasing at this instant. [4]

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Question 8 **[Maximum mark: 6]**

ball is thrown upwards. Its height above the ground, h metres, after t seconds is modelled by

$$h(t) = -4.9t^2 + 22t + 1.5.$$

(a) Find the maximum height of the ball and the time at which it is reached. [2]

(b) Find the time at which the ball hits the ground. [2]

(c) State one limitation of this model. [2]

■ Question 9 **[Maximum mark: 8]**

onsider the equation $z^2 - 4z + 13 = 0$.

(a) Solve the equation, giving your answers in the form $a + bi$. [2]

(b) Find the modulus and the argument of the root $2 + 3i$. [2]

(c) The two roots and the origin are plotted on an Argand diagram. Find the area of the triangle that they form. [2]

(d) Write the root $2 + 3i$ in Euler form $re^{i\theta}$. [2]

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Question 10 **[Maximum mark: 9]**

plane transformation is represented by the matrix

$$M = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}.$$

(a) Find the image of the point $(2, 4)$ under this transformation. [2]

(b) Find $\det(M)$ and state the area scale factor of the transformation. [2]

(c) A triangle has area 5 square units. Find the area of its image under this transformation. [2]

(d) Find M^{-1}, and hence find the point whose image under the transformation is $(7, 4)$. [3]

Question 11 **[Maximum mark: 7]**

quantity y satisfies the differential equation

$$\frac{dy}{dx} = 2x(y+1), \qquad y = 0 \text{ when } x = 0.$$

(a) Solve the differential equation to find y as a function of x. [5]

(b) Hence find the value of y when $x = 1.5$. [2]

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■ Question 12 **[Maximum mark: 8]**

he speed, v m s^{-1}, of a car is recorded every 2 seconds for the first 10 seconds of a journey.

t (s)	0	2	4	6	8	10
v (m s^{-1})	0	8	18	26	30	31

(a) Use the trapezoidal rule with all six values to estimate the distance travelled in the first 10 seconds. [3]

(b) The speed is also modelled by $v(t) = 31\left(1 - e^{-0.25t}\right)$. Use this model to find the distance travelled in the first 10 seconds. [3]

(c) Using the model, find the acceleration of the car when $t = 4$. [2]

Question 13 **[Maximum mark: 7]**

screening test is used for a condition that is present in 2% of a population. If a person has the condition, the test is positive with probability 0.95. If a person does not have the condition, the test is positive with probability 0.10.

(a) Find the probability that a randomly chosen person tests positive. [2]

(b) Given that a person tests positive, find the probability that they actually have the condition. [3]

(c) Comment on the usefulness of the test, with reference to your answer to part (b). [2]

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Question 14 **[Maximum mark: 13]**

firm's monthly profit, P thousand dollars, when it produces and sells x thousand items, is modelled by

$$P(x) = -x^3 + 9x^2 + 21x - 5, \qquad 0 \leq x \leq 10.$$

(a) (i) Find the profit when $x = 2$. [2]

(ii) Find the production level that maximises the profit, and state the maximum profit. [4]

(b) Find $P'(3)$ and explain what the sign of your answer tells you about the profit at this production level. [3]

(c) Find the values of x for which the profit is increasing. [2]

(d) State one limitation of using this cubic model when x becomes large. [2]

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End of Mock Exam 3