IB Demystified

Mathematics: Applications and Interpretation
Higher Level

Paper 2 — Mock Examination

Mock Exam 1

Graphic display calculator required
Time allowed: 2 hours
Maximum mark: 110

Information for candidates

- Do not open this examination paper until you are permitted to do so.
- A graphic display calculator is needed for every part of this paper.
- A clean copy of the *Mathematics: Applications and Interpretation HL* formula booklet may be used.
- Answer all of the questions.
- Write your solutions in the answer booklet provided. Begin each question on a fresh page.
- Unless a question directs otherwise, give numerical answers either exactly or rounded to three significant figures.
- Where a question concerns an amount of money, give your answer to the nearest cent (two decimal places) unless told otherwise.
- Any result taken from a calculator must be supported by suitable working; an unsupported answer may not earn full marks.
- When a graph is used to solve a problem, include a sketch or a clear description of the graphing method.
- The maximum mark for this paper is **110**. The time allowed is **2 hours**.

Candidate information

Candidate name: _____

Session number: _____

Date: _____

Answer all questions. Full marks are not necessarily awarded for a correct answer with no working. Solutions found from a graphic display calculator should be supported by suitable working.

1. [Maximum mark: 16]

A health clinic recorded the waiting time, t minutes, of each of 80 patients on one morning. The results are summarised in the following grouped frequency table.

Waiting time (t min)	$0 \leq t < 5$	$5 \leq t < 10$	$10 \leq t < 15$	$15 \leq t < 20$	$20 \leq t < 25$	$25 \leq t < 30$
Number of patients	8	17	23	18	9	5

(a) Write down the modal class. [1]

(b) Using the midpoint of each interval, estimate the mean and the standard deviation of the waiting times. [3]

(c) A patient whose waiting time is at least 20 minutes is said to have had a *long* wait. Estimate the probability that a randomly chosen patient had a long wait. [2]

(d) Two of the 80 patients are selected at random, one after the other, without replacement.

(i) Find the probability that both patients had a long wait. [3]

(ii) Find the probability that exactly one of the two patients had a long wait. [3]

(e) A second clinic, Clinic B, recorded the waiting times of its patients on the same morning. The five–number summary for Clinic B was

$$\text{minimum } 1, \quad Q_1 = 8, \quad \text{median } 16, \quad Q_3 = 24, \quad \text{maximum } 38 \quad \text{(minutes)}.$$

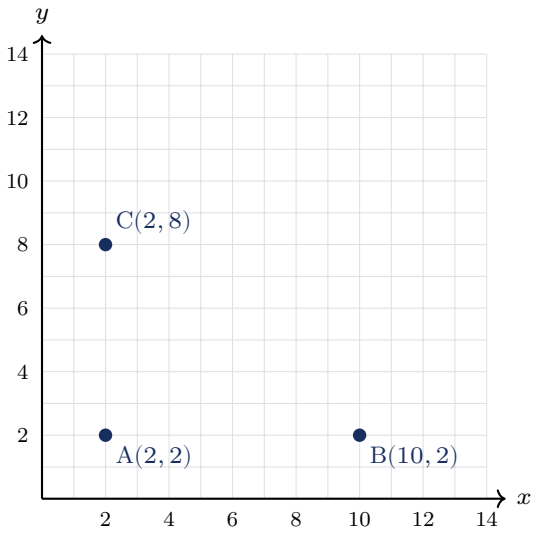
Using the median and the interquartile range of each clinic, compare the waiting times at the two clinics. [4]

2. [Maximum mark: 20]

A town monitors air quality using three sensors, A, B and C. Their positions, in kilometres, on a coordinate grid are

$$\text{A}(2,\ 2), \qquad \text{B}(10,\ 2), \qquad \text{C}(2,\ 8).$$

The sensors are shown on the grid below.



(a) Find the equation of the perpendicular bisector of [AB]. [3]

(b) Show that the perpendicular bisector of [BC] has equation $y = \frac{4}{3}x - 3$. [3]

(c) Hence find the coordinates of the point V that is equidistant from the three sensors. [2]

(d) Find the distance from V to sensor A. [2]

(e) A fourth sensor, D, is later installed at (11, 7). A pollution reading is taken at the point P(8, 6). Determine which of the four sensors is closest to P. [3]

(f) The town council wishes to install a warning siren at the position, within the monitored area, that is *as far as possible* from each of the three original sensors A, B and C. With reference to the Voronoi diagram of A, B and C, explain why this position is the point V, and state how far the siren would be from each sensor. [3]

(g) Find the area of the triangle with vertices A, B and C, and state one limitation of modelling the sensors as points on a coordinate grid. [4]

3. [Maximum mark: 19]

A new streaming service records the number of subscribers, y thousand, at the end of week x after its launch. The data for the first 16 weeks are shown in the table.

Week, x	2	4	6	8	10	12	14	16
Subscribers, y (thousand)	5	11	24	47	78	102	118	125

(a) State, with a reason, why a logistic model is more appropriate than an exponential model for these data. [2]

(b) The number of subscribers is modelled by

$$y = \frac{L}{1 + C\,e^{-kx}}.$$

Use technology to find the values of L, C and k. [4]

(c) Write down the value of the coefficient of determination R^2, and interpret it in this context. [2]

(d) Interpret the value of L in the context of the streaming service. [1]

(e) Use the model to estimate the number of subscribers at the end of week 20, and comment on the reliability of this estimate. [3]

(f) Find the week during which the number of subscribers is increasing most rapidly, and find the maximum rate of increase, in thousands of subscribers per week. [5]

(g) Give one limitation of using this model to predict the number of subscribers far into the future. [2]

4. [Maximum mark: 16]

Two delivery drones, P and Q, fly with constant velocity. Relative to an origin O, their position vectors, in metres, at time t seconds ($t \geq 0$) are

$$\mathbf{r}_{\mathrm{P}} = \begin{pmatrix} 10 \\ 0 \\ 20 \end{pmatrix} + t \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \qquad \mathbf{r}_{\mathrm{Q}} = \begin{pmatrix} 50 \\ 60 \\ 5 \end{pmatrix} + t \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}.$$

(a) Find the speed of drone P. [2]

(b) Find the acute angle between the flight paths of the two drones. [3]

(c) Show that the displacement vector from P to Q at time t is

$$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 40 - 3t \\ 60 - 6t \\ -15 + 2t \end{pmatrix},$$

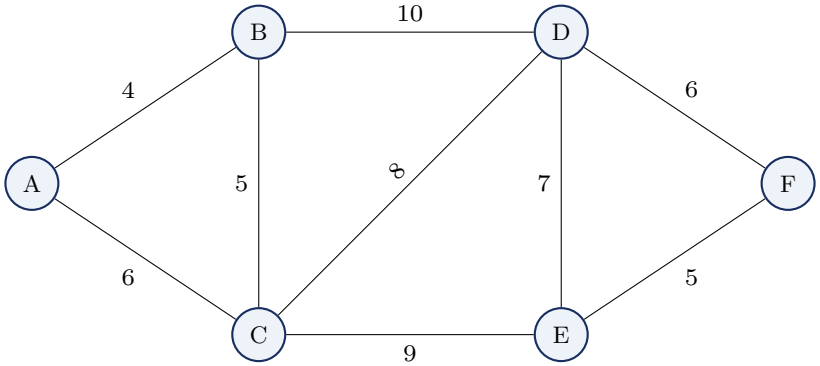
and hence find the time at which the two drones are closest together. [5]

(d) Find the minimum distance between the two drones. [3]

(e) The drones are considered to be at risk of collision if they come within 5 metres of each other. State, with a reason, whether the two drones are at risk of collision, and give one assumption made by this model. [3]

5. [Maximum mark: 21]

A courier company operates between six towns, A, B, C, D, E and F. The roads connecting the towns, and their lengths in kilometres, are shown in the network below.



(a) Considering the network as an unweighted graph, write down the adjacency matrix, and hence find the number of distinct walks of length 3 from A to F. [4]

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network. List the edges in the order they are selected, and state the total length of the tree. [5]

(c) Starting at town A, use Prim's algorithm to verify the total length of the minimum spanning tree found in part (b). [2]

(d) Find the length of the shortest route from A to F, and state this route. [4]

(e) The company wants to send a van that starts at A, visits every town and returns to A. By deleting vertex A, find a lower bound for the length of the shortest such route. [4]

(f) Give one reason why this lower bound may not equal the length of the actual shortest delivery route used. [2]

6. [Maximum mark: 18]

The number of loaves of bread sold each day by a bakery is modelled by a normal distribution with mean $\mu = 420$ loaves and standard deviation $\sigma = 35$ loaves.

(a) Find the probability that, on a randomly chosen day, the bakery sells more than 450 loaves. [2]

(b) Find the probability that the bakery sells between 380 and 450 loaves on a randomly chosen day. [2]

(c) The bakery calls a day a *busy day* if it sells more than 460 loaves. Find the probability that a randomly chosen day is a busy day. [2]

(d) In a particular month there are 30 trading days. Assuming days are independent, find the probability that there are at least 5 busy days in the month. [4]

(e) Over a longer period the owner records the number of loaves sold on 40 randomly chosen days and finds a sample mean of 425 loaves. Assuming the standard deviation of daily sales is 35 loaves, construct a 95% confidence interval for the true mean number of loaves sold per day. [4]

(f) The owner claims that the true mean number of loaves sold per day is 420. State, with a reason, whether the confidence interval in part (e) supports this claim, and state one assumption required for the binomial model used in part (d). [4]

End of examination