



IB Demystified

Mathematics: Applications and Interpretation Higher Level

Paper 2 — Mock Examination

Mock Exam 2

Graphic display calculator required

Time allowed: 2 hours

Maximum mark: 110

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Information for candidates

- Do not open this examination paper until you are permitted to do so.
- A graphic display calculator is needed for every part of this paper.
- A clean copy of the *Mathematics: Applications and Interpretation HL* formula booklet may be used.
- Answer all of the questions.
- Write your solutions in the answer booklet provided. Begin each question on a fresh page.
- Unless a question directs otherwise, give numerical answers either exactly or rounded to three significant figures.
- Where a question concerns an amount of money, give your answer to the nearest cent (two decimal places) unless told otherwise.
- Any result taken from a calculator must be supported by suitable working; an unsupported answer may not earn full marks.
- The maximum mark for this paper is **110**. The time allowed is **2 hours**.

Candidate information

Candidate name: _____

Session number: _____

Date: _____

1. [Maximum mark: 16]

Maya buys a car priced at \$24 000. She is offered two ways of financing the purchase, each repaid by equal monthly payments at the end of each month over 5 years.

- **Plan A:** borrow the full \$24 000 at a nominal annual interest rate of 6.9%, compounded monthly.
- **Plan B:** pay a deposit of 15% of the price, then borrow the remainder at a nominal annual interest rate of 5.4%, compounded monthly.

- (a) For Plan A, find the monthly payment. [3]
- (b) For Plan A, find the total amount of interest Maya would pay over the 5 years. [3]
- (c) For Plan B, find the deposit, the amount borrowed, and the monthly payment. [4]
- (d) Determine which plan is cheaper in total, and by how much. [3]
- (e) For Plan A, find the amount Maya still owes immediately after she has made 24 monthly payments. [3]

2. [Maximum mark: 18]

A hot drink is left to cool in a room kept at 22°C . Its temperature T , in $^{\circ}\text{C}$, is recorded every 5 minutes.

Time, t (min)	0	5	10	15	20	25
Temperature, T ($^{\circ}\text{C}$)	88	66	51	41	34	30

The temperature is modelled by $T = 22 + Be^{-kt}$, where B and k are positive constants.

- (a) Explain why, for this model, the quantity $T - 22$ should decrease exponentially with time. [2]
- (b) By letting $Y = \ln(T - 22)$, the model becomes linear in t . Use linear regression on the transformed data to find the values of k and B . [5]
- (c) Write down the value of the coefficient of determination R^2 for the linear fit, and comment on it. [2]
- (d) Use the model to estimate the temperature of the drink after 12 minutes, and find the time at which the temperature first falls to 30°C . [3]
- (e) Find the rate at which the temperature is decreasing 5 minutes after the drink is left to cool. [3]
- (f) Interpret the value 22 in the model, and give one limitation of the model. [3]

3. [Maximum mark: 20]

A streaming platform surveys 220 viewers about their favourite film genre. The results, classified by age group, are shown in the contingency table below.

	Action	Drama	Comedy	Total
Under 25	40	18	22	80
25–45	30	35	25	90
Over 45	10	27	13	50
Total	80	80	60	220

A χ^2 test for independence is carried out at the 5% significance level.

- (a) State the null and alternative hypotheses. [2]
- (b) Show that the expected frequency for viewers who are Under 25 and prefer Action is 29.1 (3 s.f.). [2]
- (c) Write down the number of degrees of freedom. [1]
- (d) Find the χ^2 statistic and the p -value. [3]
- (e) State, with justification, the conclusion of the test. [3]

The platform also claims that 40% of all viewers prefer Action films. In a separate random sample of 150 viewers, 48 said they prefer Action films. A test is carried out, at the 5% level, to decide whether the true proportion who prefer Action is *less* than 40%.

- (f) State suitable hypotheses for this test in terms of the proportion p . [2]
- (g) Assuming $p = 0.40$, the number who prefer Action in the sample is $X \sim B(150, 0.40)$. Find $P(X \leq 48)$. [3]
- (h) State the conclusion of this test at the 5% level, and explain what a Type I error would mean in this context. [4]

4. [Maximum mark: 17]

In a city, commuters choose between two ride-share apps, RideNow and GoTrip. Each week, behaviour is modelled as follows.

- A RideNow user remains with RideNow with probability 0.85, and switches to GoTrip with probability 0.15.
- A GoTrip user remains with GoTrip with probability 0.75, and switches to RideNow with probability 0.25.

At the start of week 0 there are 4000 RideNow users and 6000 GoTrip users. The state of the system is written as a column vector $\begin{pmatrix} r \\ g \end{pmatrix}$, where r and g are the numbers using RideNow and GoTrip.

- Write down the transition matrix T for this model, and interpret the entry 0.85. [2]
- Find the number of users of each app after one week. [3]
- Find the number of users of each app after three weeks. [3]
- Find the long-term (steady-state) number of users of each app. [4]
- By writing the steady state as a system of equations, verify your answer to part (d). [3]
- State one assumption of this Markov model that may not hold in reality. [2]

5. [Maximum mark: 19]

A drinks company manufactures a closed cylindrical can of radius r cm and height h cm. Each can must hold exactly 330 cm^3 of liquid.

- (a) Show that the total surface area, $S \text{ cm}^2$, of a can is given by

$$S = 2\pi r^2 + \frac{660}{r}.$$

[3]

- (b) Find $\frac{dS}{dr}$ and hence find the value of r that minimises the surface area. [4]

- (c) Find the minimum surface area of a can. [3]

- (d) The metal used to make the cans costs $\$0.0015$ per cm^2 . Find the minimum cost of the metal for one can. [2]

- (e) For a premium can, the metal used for the top and bottom costs twice as much per cm^2 as the metal used for the curved side. Taking the side metal to cost $\$0.0015$ per cm^2 , find the radius that minimises the total cost of a premium can. [5]

- (f) State one practical reason why a real drinks can may not use these dimensions. [2]

6. [Maximum mark: 20]

A small business records its monthly advertising spend, x (in thousands of dollars), and the number of units sold that month, y , for eight months.

Advertising spend, x	2	3	5	6	8	9	11	12
Units sold, y	22	28	41	47	58	60	71	75

- (a) Find the equation of the regression line of y on x , and the value of Pearson's correlation coefficient r . [4]
- (b) Interpret the value of r , and the value of the coefficient of determination R^2 , in this context. [2]
- (c) Use your regression line to estimate the number of units sold in a month with an advertising spend of \$7000. [2]
- (d) Find the residual for the month in which the advertising spend was \$8000. [3]
- (e) Find Spearman's rank correlation coefficient for these data. [4]
- (f) Comparing r and Spearman's coefficient, state which is the more appropriate measure here, giving a reason, and explain why predicting sales for an advertising spend of \$20 000 would be unreliable. [5]

End of examination