IB Demystified

Mathematics: Applications and Interpretation
Higher Level

Paper 2 — Mock Examination

Mock Exam 3

Graphic display calculator required
Time allowed: 2 hours
Maximum mark: 110

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Information for candidates

- Do not open this examination paper until you are permitted to do so.
- A graphic display calculator is needed for every part of this paper.
- A clean copy of the *Mathematics: Applications and Interpretation HL* formula booklet may be used.
- Answer all of the questions.
- Write your solutions in the answer booklet provided. Begin each question on a fresh page.
- Unless a question directs otherwise, give numerical answers either exactly or rounded to three significant figures.
- Where a question concerns an amount of money, give your answer to the nearest cent (two decimal places) unless told otherwise.
- Any result taken from a calculator must be supported by suitable working; an unsupported answer may not earn full marks.
- The maximum mark for this paper is **110**. The time allowed is **2 hours**.

Candidate information

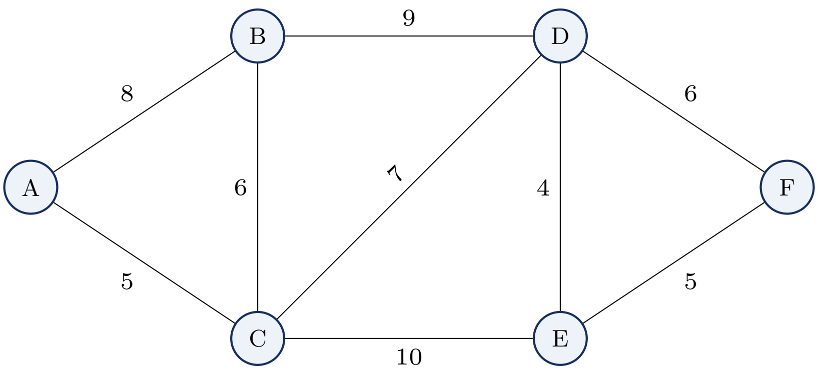
Candidate name: ______________________________

Session number: ______________________________

Date: ______________________________

1. [Maximum mark: 18]

A street-cleaning vehicle must travel along every road in a district and return to its depot at town A. The road network connecting the six towns A–F is shown below, with road lengths in kilometres.



(a) Write down the degree of each vertex, and identify the vertices of odd degree. [3]

(b) Explain why this network does not have an Eulerian circuit. [2]

(c) Find the shortest path between the two odd-degree vertices, and state its length. [3]

(d) Hence find the length of the shortest closed route that travels along every road at least once and returns to A. [3]

(e) State which road or roads must be travelled twice on this route. [2]

(f) Find the shortest route from A to F, and state its length. [3]

(g) Give one reason why this model may not give the route a real vehicle should follow. [2]

2. [Maximum mark: 19]

A factory produces metal rods. The length, L mm, of a rod is normally distributed. It is known that 5% of rods are shorter than 98 mm and 10% of rods are longer than 104 mm.

(a) Find the mean μ and the standard deviation σ of L. [5]

(b) Find $P(99 < L < 103)$. [2]

(c) A rod is rejected if $L < 98$ or $L > 104$. Find the probability that a randomly chosen rod is rejected. [2]

(d) A box contains 20 rods, chosen at random. Find the probability that at most 2 rods in the box are rejected. [3]

(e) For the same box of 20 rods, find the probability that exactly 2 are rejected, given that at least 1 is rejected. [3]

(f) The mean length of a random sample of 25 rods is recorded. Find the probability that this sample mean exceeds 101 mm. [4]

3. [Maximum mark: 17]

Let $z_1 = 3 + 4\mathrm{i}$ and $z_2 = 1 - 2\mathrm{i}$.

(a) Find $z_1 z_2$ and $\dfrac{z_1}{z_2}$, giving each answer in the form $a + b\mathrm{i}$. [4]

(b) Find the modulus $|z_1|$ and the argument $\arg(z_1)$, in radians. [3]

(c) Write z_1 in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$. [2]

(d) Find z_1^3, giving your answer in the form $a + b\mathrm{i}$. [3]

In an alternating-current circuit, two voltages are represented by the complex numbers

$$V_1 = 5\,\mathrm{e}^{0.3\mathrm{i}} \qquad \text{and} \qquad V_2 = 3\,\mathrm{e}^{1.1\mathrm{i}},$$

where the angles are in radians. The combined voltage is $V_1 + V_2$.

(e) Find the combined voltage $V_1 + V_2$, giving your answer in the form $R\,\mathrm{e}^{\mathrm{i}\varphi}$ with $R > 0$ and $0 \le \varphi < 2\pi$. [5]

4. [Maximum mark: 18]

An invasive plant species is spreading across a wetland. The area it covers, A hectares, t years after it is first observed, is modelled by the differential equation

$$\frac{\mathrm{d}A}{\mathrm{d}t} = 0.5A - 0.01A^2.$$

When first observed ($t = 0$), the species covers 10 hectares.

(a) Find the equilibrium values of A (the values for which $\frac{\mathrm{d}A}{\mathrm{d}t} = 0$). [3]

(b) State which equilibrium value represents the carrying capacity, and interpret it in context. [2]

(c) Use Euler's method with a step length of $h = 0.5$ years to estimate the area covered after 2 years. [5]

(d) Sketch, in words, the long-term behaviour of A for a solution that starts at $A = 10$, and state what happens to $\frac{\mathrm{d}A}{\mathrm{d}t}$ as A approaches the carrying capacity. [3]

(e) Find the area at which the species is spreading most rapidly, and find this maximum rate of spread. [4]

(f) Give one limitation of using this model. [1]

5. [Maximum mark: 18]

A solar-panel array is mounted as a flat triangular surface with corners at the points $\mathrm{P}(1,2,3)$, $\mathrm{Q}(5,4,2)$ and $\mathrm{R}(3,6,5)$, where distances are in metres.

(a) Find $\overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{PR}}$. [2]

(b) Find $\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}$, and hence find the area of the triangular panel PQR. [4]

(c) Find the Cartesian equation of the plane Π containing P, Q and R. [4]

(d) A support cable lies along the line $L: \ \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$. Find the acute angle between the cable and the plane Π. [4]

(e) Find the shortest distance from the point $\mathrm{S}(4,4,9)$ to the plane Π. [4]

6. [Maximum mark: 20]

A graphic designer transforms a logo on screen. The logo is the triangle with vertices $A(2,1)$, $B(6,1)$ and $C(2,4)$. All transformations are about the origin.

(a) Write down the matrix $\mathbf{R}$ representing an anticlockwise rotation of 90° about the origin, and find the image of the point A under this rotation. [3]

(b) The matrix $\mathbf{S} = \begin{pmatrix} 1.5 & 0 \\ 0 & 1.5 \end{pmatrix}$ is applied to the logo. Using the determinant of $\mathbf{S}$ as an area scale factor, find the area of the image triangle. [4]

(c) Reflection in the line $y = x$ is represented by the matrix $\mathbf{F} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the matrix product $\mathbf{FR}$ (the rotation followed by the reflection), and describe fully the single transformation it represents. [5]

(d) Find the determinant of $\mathbf{FR}$, and interpret its value geometrically. [2]

(e) The designer wants a single matrix $\mathbf{M}$ that maps $A(2,1) \to (1,2)$ and $B(6,1) \to (1,6)$. Find $\mathbf{M}$ and describe the transformation it represents. [4]

(f) Explain, using its determinant, why the transformation with matrix $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ cannot be undone (has no inverse). [2]

End of examination