IB Demystified

Examiners • Moderators • Mentors

Mathematics: Applications and Interpretation
Higher Level

Paper 3 — Mock Examination

Mock Exam 1

Graphic Display Calculator Required

Time allowed: **1 hour 15 minutes** (75 minutes)

Maximum mark: **55 marks**

Instructions to Candidates

- A graphic display calculator is needed throughout this paper.
- You may use a clean copy of the *Mathematics: Applications and Interpretation HL* formula booklet.
- There are two questions. Answer **both** of them.
- Write your responses in the answer booklet or on the continuation pages provided.
- Begin each question on a fresh page.
- Unless a question states otherwise, give numerical answers either exactly or rounded to three significant figures.
- Any result obtained from a calculator must be accompanied by suitable supporting working.
- Where a graph is used to reach an answer, include a sketch or a clear description of the graphical method.
- Answers presented without supporting working may not earn full marks.
- The maximum mark for this paper is **55**. The time allowed is **1 hour 15 minutes**.

Candidate information

Candidate name: _____

Session number: _____

Date: _____

1. [Maximum mark: 29]

This question investigates how subscribers move between three competing music-streaming platforms, and how the revenue of one platform might be modelled over time.

Three platforms — **Nimbus**, **Vortex** and **Cascade** — compete in the same market. Each month a subscriber keeps their current platform or switches to one of the other two. Market researchers model this monthly movement with the transition matrix T, where the entry in row i, column j gives the probability that a subscriber currently using platform i will be using platform j in the following month.

$$T = \begin{array}{c} \\ \text{N} \\ \text{V} \\ \text{C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{N} & \text{V} & \text{C} \end{array} \\ \begin{pmatrix} 0.80 & 0.12 & 0.08 \\ 0.10 & 0.78 & 0.12 \\ 0.09 & 0.06 & 0.85 \end{pmatrix} \end{array}$$

At the start of the study (month 0) the platforms hold 50%, 30% and 20% of the market respectively. This is written as the row vector $\mathbf{s}_0 = (\,0.50 \quad 0.30 \quad 0.20\,)$.

(a) Interpret, in context, the value 0.12 in row 1, column 2 of T. **[2]**

(b) Show that the market shares after one month are $(\,0.448 \quad 0.306 \quad 0.246\,)$, and state which platform has gained the most market share in that month. **[3]**

(c) (i) Find T^3. **[2]**

(ii) Hence determine the probability that a subscriber using Nimbus in month 0 is using Cascade in month 3. **[2]**

(d) (i) Find the long-term (steady-state) market shares of the three platforms. **[4]**

(ii) Explain what is meant by a steady-state vector in this context. **[1]**

This question continues on the following page

(Question 1 continued)

Nimbus plans an advertising campaign to improve customer loyalty. The campaign is expected to raise the monthly retention probability of Nimbus (currently 0.80) to a new value r. Any subscribers who still leave Nimbus are assumed to move to Vortex and Cascade in the ratio $3 : 2$.

(e) Find the value of r for which the long-term market share of Nimbus would become exactly 40%. **[5]**

The Nimbus finance team models the platform's monthly revenue, in millions of dollars, using two different models. In each model, month 1 has a revenue of 4.00 million dollars.

- **Model A** (arithmetic): revenue increases by a fixed amount of 0.35 million dollars each month.
- **Model B** (geometric): revenue increases by a fixed factor of 6% each month.

(f) (i) Write down an expression for the revenue in month n under each model. **[2]**

(ii) Find the first month in which Model B predicts a greater revenue than Model A. **[3]**

(iii) Find the total revenue predicted by each model over the first 12 months. **[2]**

(g) The two models give very different long-term predictions. State one limitation of using the geometric model (Model B) to forecast Nimbus revenue several years ahead, and one limitation of assuming the transition matrix T stays constant over time. **[3]**

[Total: 29 marks]

2. [Maximum mark: 26]

This question uses statistical models and hypothesis tests to investigate the aerobic fitness of athletes, measured by VO_2 max in millilitres of oxygen per kilogram per minute ($\text{ml kg}^{-1}\text{min}^{-1}$).

In a large national population of recreational runners, VO_2 max is modelled by a normal distribution with mean $\mu = 45$ and standard deviation $\sigma = 6$.

(a) (i) Find the probability that a randomly chosen runner has a VO_2 max greater than 50. **[2]**

(ii) Find the probability that a randomly chosen runner has a VO_2 max between 40 and 52. **[2]**

(b) A coach selects a random sample of 9 runners and records the sample mean VO_2 max, $\bar{X}$.

(i) State the distribution of $\bar{X}$, giving its mean and standard deviation. **[2]**

(ii) Find $P(\bar{X} > 47)$. **[2]**

A sports scientist runs a six-week training programme. The VO_2 max of 10 athletes is recorded before and after the programme, as shown in Table 1.

Table 1

Athlete	1	2	3	4	5	6	7	8	9	10
Before	42.1	38.6	45.3	40.0	47.2	43.5	39.8	44.6	41.0	46.3
After	44.8	41.0	46.9	43.2	48.0	46.1	42.7	46.0	44.3	48.5

(c) The scientist wishes to test, at the 5% significance level, whether the programme increases VO_2 max. Stating your hypotheses clearly, carry out an appropriate test and give your conclusion in context. **[6]**

(d) Construct a 95% confidence interval for the mean increase in VO_2 max produced by the programme, and interpret it in context. **[4]**

This question continues on the following page

(Question 2 continued)

To compare two supplements, the scientist records the VO_2 max *improvement* of 10 athletes taking an active supplement and 10 athletes taking a placebo. The unbiased sample variances of the improvements are

$$s^2_{\text{supplement}} = 0.405, \qquad s^2_{\text{placebo}} = 0.808.$$

(e) The scientist intends to use a two-sample t-test that assumes equal population variances.

(i) Carry out an F-test, at the 5% significance level, to decide whether the assumption of equal variances is reasonable. State your hypotheses and your conclusion. **[4]**

(ii) State whether the equal-variance assumption is supported. **[1]**

(f) State **two** assumptions required for the paired t-test in part (c) to be valid, and explain one practical reason why the conclusion of that test might not generalise to all recreational runners. **[3]**

[Total: 26 marks]