



IB Demystified

Examiners • Moderators • Mentors

Mathematics: Applications and Interpretation Higher Level

Paper 3 — Mock Examination

Mock Exam 2

Graphic Display Calculator Required

Time allowed: **1 hour 15 minutes** (75 minutes)

Maximum mark: **55 marks**

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Instructions to Candidates

- A graphic display calculator is needed throughout this paper.
- You may use a clean copy of the *Mathematics: Applications and Interpretation HL* formula booklet.
- There are two questions. Answer **both** of them.
- Write your responses in the answer booklet or on the continuation pages provided.
- Begin each question on a fresh page.
- Unless a question states otherwise, give numerical answers either exactly or rounded to three significant figures.
- Monetary answers should be given to two decimal places unless stated otherwise.
- Any result obtained from a calculator must be accompanied by suitable supporting working.
- Where a graph is used to reach an answer, include a sketch or a clear description of the graphical method.
- Answers presented without supporting working may not earn full marks.
- The maximum mark for this paper is **55**. The time allowed is **1 hour 15 minutes**.

Candidate information

Candidate name: _____

Session number: _____

Date: _____

This question investigates the financial decisions facing the owner of a small café who must buy a new espresso machine, and compares borrowing with saving.

Mara owns a café and needs a commercial espresso machine costing \$18 000. She borrows the full amount as a loan that charges a nominal annual interest rate of 6.9%, compounded monthly. She agrees to repay the loan with equal monthly payments over 4 years.

- (a) Find Mara's monthly repayment. [3]
- (b) Find the amount still owing on the loan immediately after she has made 24 payments (that is, after 2 years). [3]
- (c) Find the total interest Mara will pay over the full 4 years. [2]

After exactly 2 years (with the balance from part (b) still owing), the lender raises the nominal annual interest rate to 8.4%, compounded monthly. Mara keeps the original end date, so she repays the outstanding balance with equal monthly payments over the remaining 2 years.

- (d) Find Mara's new monthly repayment for the final 2 years, and hence the total interest she pays over the whole loan under this revised plan. [5]
- (e) As an alternative, Mara could instead have taken a 6-year loan at the original 6.9% rate. Find the monthly repayment and the total interest for this 6-year plan, and state, with a reason, which feature of this plan a borrower might prefer and which they might not. [4]

This question continues on the following page

(Question 1 continued)

Separately, Mara sets up a savings plan for future equipment. She deposits \$250 at the end of each month into an account paying a nominal annual interest rate of 5.4%, compounded monthly.

- (f) (i) Find the value of the savings account after 5 years. **[3]**
(ii) Find the number of complete months required for the savings account to first exceed \$20 000. **[3]**

The new espresso machine is expected to raise the café's monthly profit. In its first month it adds \$520 of extra profit, and this extra monthly profit then increases by a fixed \$15 each month, forming an arithmetic sequence.

- (g) (i) Taking the total cost of the original 4-year loan (from part (c) plus the \$18 000 principal repaid) as the amount to be recovered, find the first month in which the cumulative extra profit first exceeds this total cost. **[4]**
(ii) State one reason why this break-even calculation may be over-optimistic. **[1]**

[Total: 28 marks]

This question investigates how to choose a suitable model for the growth in the number of active users of a new mobile app, and how reliable that model is.

A start-up records the number of active users (in thousands) of its app at the end of each of the first 12 months after launch. The data are shown in Table 1.

Table 1

Month t	1	2	3	4	5	6	7	8	9	10	11	12
Users (thousands)	7.8	14.3	24.9	41.7	68.1	99.8	140	180	196	212	230	235

(The month-8 to month-12 values are 180.2, 195.8, 211.9, 229.7, 235.0 to one decimal place.)

- (a) (i) Find the equation of the least-squares regression line of users on month, and write down Pearson's correlation coefficient r and the coefficient of determination R^2 . **[3]**
- (ii) State what the value of r alone might suggest about the relationship. **[1]**
- (b) Despite the high value of R^2 , explain why a linear model is **not** suitable for these data. Refer to the behaviour of the residuals or the shape of the data. **[2]**
- (c) A colleague suggests an exponential model of the form $y = ab^t$. Explain why an exponential model also fails to describe the later months well. **[3]**

This question continues on the following page

(Question 2 continued)

- (d) A logistic model of the form

$$y = \frac{L}{1 + C e^{-kt}}$$

is proposed.

- (i) Use technology to find the values of L , C and k , and state the coefficient of determination.
[4]
 - (ii) Interpret the value of L in the context of the app. **[2]**
- (e) For the logistic model, the rate of growth (in thousands of users per month) is greatest at a particular time.
- (i) Find the month t at which the growth rate is greatest. **[3]**
 - (ii) Find the greatest growth rate, in thousands of users per month. **[2]**
- (f) Use the logistic model to predict the number of active users at the end of month 18, and comment on how reliable this prediction is likely to be. **[3]**
- (g) State, with reasons, which of the three models the start-up should use to describe the first year, and one reason why even the best model should be used cautiously for long-term planning. **[2]**

[Total: 27 marks]