IB Demystified

Examiners • Moderators • Mentors

Mathematics: Applications and Interpretation Higher Level

Paper 3 — Mock Examination

Mock Exam 3

Graphic Display Calculator Required

Time allowed: **1 hour 15 minutes** (75 minutes)

Maximum mark: **55 marks**

Instructions to Candidates

- A graphic display calculator is needed throughout this paper.
- You may use a clean copy of the *Mathematics: Applications and Interpretation HL* formula booklet.
- There are two questions. Answer **both** of them.
- Write your responses in the answer booklet or on the continuation pages provided.
- Begin each question on a fresh page.
- Unless a question states otherwise, give numerical answers either exactly or rounded to three significant figures.
- Any result obtained from a calculator must be accompanied by suitable supporting working.
- Where a graph is used to reach an answer, include a sketch or a clear description of the graphical method.
- Answers presented without supporting working may not earn full marks.
- The maximum mark for this paper is **55**. The time allowed is **1 hour 15 minutes**.

Candidate information

Candidate name: ______________________________

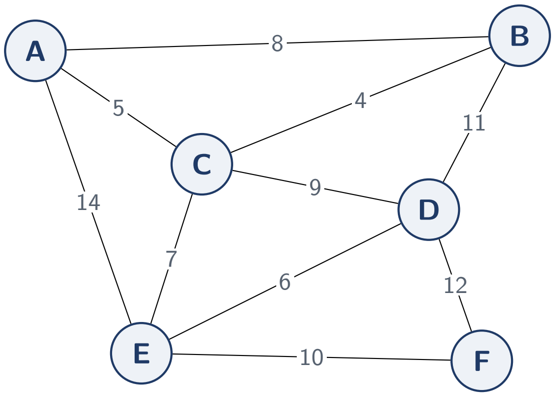
Session number: ______________________________

Date: ___________________________________

1. [Maximum mark: 27]

This question investigates the design of a community fibre-optic network connecting six towns, using a graph in which vertices represent towns and edges represent possible cables, with weights giving the cost (in thousands of dollars) of laying each cable.

A regional council is planning a fibre-optic network for six towns, labelled A, B, C, D, E and F. The possible direct cable links and their costs (in thousands of dollars) are shown in the weighted graph below.



The information in the graph can also be written as a weight matrix $\mathbf{W}$, where the entry in row i, column j is the cost of the direct cable between the two towns (and is left blank where no direct cable exists).

(a) Write down the order of the rows and columns you will use, and state the value of the entry of $\mathbf{W}$ in the row for C and the column for E. Hence state the cost of the direct cable from C to E. **[3]**

For parts (b)–(c), ignore the costs and consider only whether a direct cable exists between each pair of towns. Let $\mathbf{M}$ be the (unweighted) adjacency matrix of the graph, where the entry is 1 if a direct cable exists and 0 otherwise.

(b) (i) Find the number of walks of length 2 from A to D. **[2]**

(ii) Find the number of walks of length 3 from A to F. **[2]**

This question continues on the following page

(Question 1 continued)

The council first wants to connect all six towns using cables, at the least possible total cost, so that there is a path (directly or indirectly) between every pair of towns.

(c) (i) Use a suitable algorithm to find a minimum spanning tree for the network. List the cables chosen, in the order in which your algorithm selects them. **[4]**

(ii) State the minimum total cost of connecting all six towns. **[1]**

Town A houses the council's main data centre, and town F houses a new hospital. The council wants data to travel from A to F along the cables at the least possible cost.

(d) Use a suitable algorithm to find the least-cost route from A to F, and state its total cost. **[5]**

For maintenance, an engineer must travel along **every** cable in the full network at least once, starting and finishing at town A. Travelling along a cable costs the same amount (in thousands of dollars) as laying it.

(e) (i) By identifying the vertices of odd degree, determine the least total cost of such a maintenance route. **[5]**

(ii) State one practical reason why the council might *not* use the minimum spanning tree from part (c) as the network actually built. **[1]**

[Total: 27 marks]

2. [Maximum mark: 28]

This question investigates the lifetimes of LED light bulbs produced by a factory, using the normal and binomial distributions, conditional probability, a confidence interval and a hypothesis test.

A factory produces LED light bulbs. The lifetime X, in hours, of a randomly chosen bulb is modelled by a normal distribution with mean 1200 hours and standard deviation 90 hours.

(a) (i) Find $P(X < 1000)$. **[2]**

(ii) Find $P(1150 < X < 1300)$. **[2]**

(b) A bulb is said to "fail early" if its lifetime is less than 1050 hours. Show that the probability that a randomly chosen bulb fails early is 0.0478, correct to three significant figures. **[2]**

Bulbs are sold in packs of 12. Assume that whether each bulb fails early is independent of the others, and let Y be the number of bulbs in a pack that fail early.

(c) (i) Find the probability that exactly one bulb in a pack fails early. **[2]**

(ii) Find the probability that at least two bulbs in a pack fail early. **[3]**

This question continues on the following page

(Question 2 continued)

Before bulbs are packed, each bulb passes through an automatic optical scanner intended to detect defective bulbs. From long-term records:

- 4% of all bulbs produced are genuinely defective;
- if a bulb is defective, the scanner flags it with probability 0.96;
- if a bulb is not defective, the scanner still flags it (a false alarm) with probability 0.05.

(d) (i) Find the probability that a randomly chosen bulb is flagged by the scanner. **[3]**

(ii) Given that a bulb is flagged by the scanner, find the probability that it is genuinely defective. Comment briefly on what your answer means for the usefulness of the scanner. **[3]**

To check the mean lifetime of current production, an inspector tests a random sample of 40 bulbs and records a sample mean of 1183 hours with sample standard deviation 96 hours.

(e) Find a 95% confidence interval for the mean lifetime of the bulbs, and state what this interval suggests about the claimed mean of 1200 hours. **[4]**

The factory develops a new filament that it believes increases the mean lifetime above 1200 hours. A random sample of 15 bulbs made with the new filament has the following lifetimes (in hours):

1245	1198	1320	1276	1188
1302	1255	1210	1288	1330
1175	1265	1294	1240	1222

(f) (i) State suitable null and alternative hypotheses for a test of whether the new filament increases the mean lifetime. **[2]**

(ii) Perform an appropriate t-test at the 5% significance level. State the test statistic, the p-value, and your conclusion in context. **[4]**

(iii) State one assumption required for this test to be valid. **[1]**

[Total: 28 marks]