IB Demystified

Examiners. Moderators. Mentors.

Mathematics: Applications and Interpretation

Standard Level

Paper 2

Mock Examination

Question Paper

Mock Exam 2

Calculator:	Graphic display calculator required
Time allowed:	1 hour 30 minutes
Maximum mark:	80 marks

Instructions to Candidates

- A graphic display calculator (GDC) is required throughout this paper.
- A clean copy of the *Mathematics: Applications and Interpretation SL* formula booklet may be used.
- Answer **all** questions in the answer booklet or on the continuation pages provided.
- Begin each question on a new page of your answer booklet.
- Unless a question states otherwise, give numerical answers exactly or correct to three significant figures.
- Solutions obtained from a calculator must be supported by suitable working, such as a clear setup, a sketch, or a description of the method used.
- Where a graph is used to reach an answer, include a sketch or a clear description of the graphing method.
- Unsupported answers may not receive full marks, so you are advised to show all working.
- The maximum mark for this paper is **80 marks**.
- The time allowed for this paper is **1 hour 30 minutes**.

Candidate Information

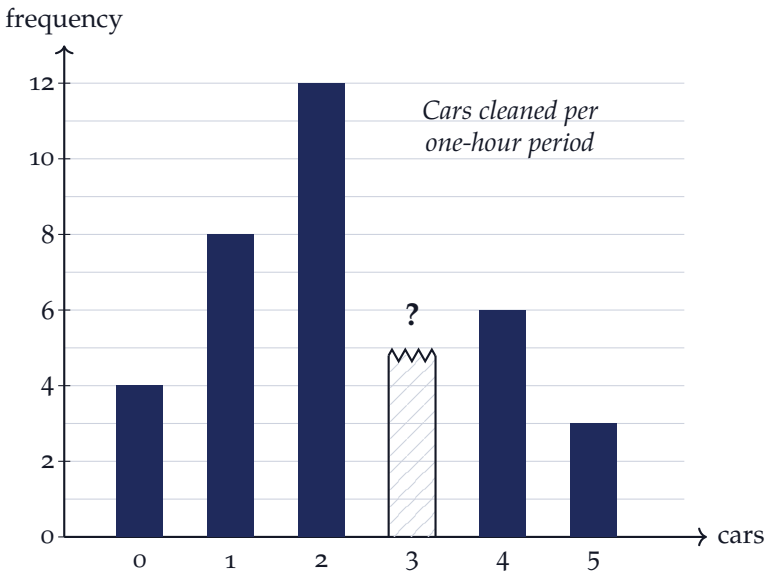
Candidate name: _____

Session number: _____

Date: _____

1. [Maximum mark: 15]

A self-service car wash records the number of cars it cleans during each one-hour period across a sample of 40 operating hours. The results are shown in the bar chart below. The height of the bar for hours in which exactly **3 cars** were cleaned has been smudged and is unreadable.



The frequency for hours in which exactly 3 cars were cleaned is denoted by k.

Cars cleaned	0	1	2	3	4	5
Frequency	4	8	12	k	6	3

(a) (i) Write down the number of hours in which exactly 4 cars were cleaned. [1]

(ii) The mean number of cars cleaned per hour is 2.3. Find the value of k. [3]

(b) Find the median and the interquartile range of the number of cars cleaned per hour. [3]

One of the 40 hours is selected at random.

(c) Find the probability that at least 3 cars were cleaned during that hour. [2]

The manager now selects 8 operating hours at random. The number of these hours in which at least 3 cars were cleaned is denoted by the random variable X. You may assume that X follows a binomial distribution with probability of success equal to your answer to part (c).

(d) (i) Find the probability that exactly 3 of these hours had at least 3 cars cleaned. [3]

(ii) Find the probability that more than 3 of these hours had at least 3 cars cleaned. [3]

Turn over ▶

2. [Maximum mark: 15]

Give all answers in this question correct to the nearest dollar, unless stated otherwise.

A printing business borrows $48\,000$ US dollars (USD) to purchase new machinery. The loan has a nominal annual interest rate of 9%, compounded monthly, and is to be repaid by equal payments at the end of each month over 6 years.

(a) Determine the monthly repayment. [3]

(b) Find the total amount of interest the business will pay over the full term of the loan. [2]

(c) Determine the amount still owing on the loan immediately after the 36th repayment has been made. [3]

Immediately after the 36th repayment, the business makes an additional one-off payment of $10\,000$ USD, which reduces the amount owing. It then continues to make the same monthly repayment as before until the loan is fully repaid.

(d) (i) Find the amount owing immediately after the one-off payment. [1]

(ii) Determine the number of further monthly repayments required to clear the loan. [3]

(e) (i) State how many months earlier the loan is repaid compared with the original 6-year schedule. [1]

(ii) Suggest one reason why, in practice, this strategy might not save the business as much as the calculation suggests. [2]

Turn over ▶

3. [Maximum mark: 17]

A workshop produces customised parts using two machines. The workshop operates for 30 consecutive days.

Machine P prints 20 parts on day 1. On each following day, the number of parts it prints is 6% greater than on the previous day.

(a) Explain why the daily numbers of parts printed by Machine P form a geometric sequence, and state the common ratio. [2]

(b) Find the number of parts Machine P prints on day 12. Give your answer to the nearest whole part. [2]

(c) Find the total number of parts Machine P prints during the 30 days. [3]

(d) Determine the first day on which Machine P prints more than 100 parts. [3]

Machine Q prints 50 parts on day 1. On each following day it prints a fixed number of parts more than on the previous day. On day 10 it prints 95 parts. The daily numbers of parts printed by Machine Q form an arithmetic sequence.

(e) (i) Find the fixed daily increase for Machine Q. [2]

(ii) Find the number of parts Machine Q prints on day 20. [1]

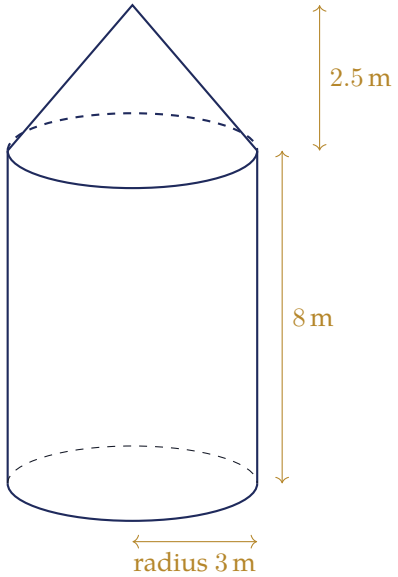
(f) (i) Find the total number of parts Machine Q prints during the 30 days. [2]

(ii) Hence state, with supporting values, which machine prints more parts in total over the 30 days. [2]

Turn over ▶

4. [Maximum mark: 14]

A grain store is modelled as a solid formed by a cylinder with a cone on top, sharing the same circular cross-section, as shown below. The cylinder has radius 3 m and height 8 m. The cone has the same radius and a height of 2.5 m. **diagram not to scale**



(**a**) Find the total volume of the grain store. [3]

(**b**) Find the angle that the slant surface of the cone makes with the horizontal. [3]

The curved surface of the cylinder and the curved surface of the cone are to be painted with a protective coating. The flat circular base is not painted.

(**c**) Find the total area to be painted. [3]

The coating is sold only in whole litres. One litre covers $6\ \text{m}^2$, and each litre costs 13.50 USD.

(**d**) Determine the cost of the coating required for one grain store. [3]

(**e**) State one reason why the amount of coating actually purchased may differ from a value found by dividing the painted area by 6. [2]

Turn over ▶

5. [Maximum mark: 19]

A logistics company operates two distribution centres, P and Q. On a coordinate grid where 1 unit represents 1 km, the centres are located at

$$P(2,\ 3) \qquad \text{and} \qquad Q(14,\ 9).$$

(a) (i) Find the distance PQ. [2]

(ii) Find the coordinates of the midpoint of $[PQ]$. [2]

(iii) A delivery boundary is drawn so that every point on it is the same distance from P as from Q. Find the equation of this boundary. [3]

The company plans to open a new sorting hub along a straight access road. When the hub is located x kilometres along the road, its daily operating cost, in hundreds of USD, is modelled by

$$C(x) = 0.5x^2 - 8x + 90, \qquad 0 \le x \le 20.$$

(b) Find the daily operating cost when $x = 0$, and state what this represents in context. [2]

(c) Using calculus, find the value of x that minimises the daily operating cost, and find this minimum cost. Justify that your value gives a minimum. [4]

(d) Find the values of x for which the modelled daily operating cost is 7000 USD. [3]

(e) The company decides to locate the hub at the position found in part (c). Explain one reason why this model may not fully represent the true operating cost of the hub. [3]

End of Mock Examination 2

Total: 80 marks