IB Demystified

Examiners. Moderators. Mentors.

Mathematics: Applications and Interpretation

Standard Level

Paper 2

Mock Examination

Question Paper

Mock Exam 3

Calculator:	Graphic display calculator required
Time allowed:	1 hour 30 minutes
Maximum mark:	80 marks

Instructions to Candidates

- A graphic display calculator (GDC) is required throughout this paper.
- A clean copy of the *Mathematics: Applications and Interpretation SL* formula booklet may be used.
- Answer **all** questions in the answer booklet or on the continuation pages provided.
- Begin each question on a new page of your answer booklet.
- Unless a question states otherwise, give numerical answers exactly or correct to three significant figures.
- Solutions obtained from a calculator must be supported by suitable working, such as a clear setup, a sketch, or a description of the method used.
- Where a graph is used to reach an answer, include a sketch or a clear description of the graphing method.
- Unsupported answers may not receive full marks, so you are advised to show all working.
- The maximum mark for this paper is **80 marks**.
- The time allowed for this paper is **1 hour 30 minutes**.

Candidate Information

Candidate name: _____

Session number: _____

Date: _____

1. [Maximum mark: 15]

At a charity fair, players pay 5 US dollars (USD) to spin a prize wheel once. The amount of money W, in USD, that a player wins on one spin has the probability distribution shown below.

Winnings w (USD)	0	2	5	20
Probability $P(W = w)$	0.55	0.25	p	0.05

(a) Find the value of p. [2]

(b) Find the expected winnings $E(W)$ for one spin. [3]

The fair keeps the difference between the entry fee and the amount paid out as profit.

(c) Show that the expected profit to the fair for one spin is 2.75 USD, and hence find the expected profit if the wheel is spun 200 times. [3]

A player decides to play the game on two separate spins. The outcomes of the two spins are independent.

(d) (i) Find the probability that the player wins the 20 USD prize on at least one of the two spins. [3]

(ii) Find the player's expected total winnings from the two spins, and state whether the player should expect to make a profit overall. [1]

(e) Explain one reason why a player who spins the wheel only once may have an experience that differs greatly from the expected value found in part (b). [3]

Turn over ▶

2. [Maximum mark: 15]

Give all monetary answers in this question correct to the nearest dollar, unless stated otherwise.

Lena is saving towards the purchase of a car. She opens a savings account that pays a nominal annual interest rate of 6%, compounded monthly.

Lena first considers placing a single deposit of 5000 USD into the account and leaving it untouched.

(a) Determine the value of this single deposit after 4 years. [3]

Lena instead decides to deposit 150 USD into the account at the end of each month.

(b) (i) Determine the value of the account after 4 years of monthly deposits. [3]

(ii) State which plan produces the greater balance after 4 years, and by how much. [1]

(c) Lena would like the monthly-deposit plan to reach $12\,000$ USD after 4 years. Determine the monthly deposit she would need to make to achieve this. [4]

(d) (i) If Lena deposits 250 USD at the end of each month, determine the number of months required for the account to first reach $20\,000$ USD. [3]

(ii) State one reason why the actual time taken might be longer than this calculated value. [1]

Turn over ▶

3. [Maximum mark: 17]

A ball is dropped vertically from a height of $2.0\,\text{m}$ onto a hard floor. After each bounce, the ball rises to a peak height equal to 65% of its previous peak height. Let h_n be the peak height, in metres, reached after the nth bounce.

(**a**) Explain why the peak heights $h_1,\ h_2,\ h_3, \ldots$ form a geometric sequence, and state the common ratio. [2]

(**b**) Find the peak height reached after the 4th bounce. [2]

(**c**) Determine the first bounce after which the peak height is less than $0.1\,\text{m}$. [3]

(**d**) Find the total of the peak heights reached during the first 10 bounces. [3]

The ball is modelled as continuing to bounce indefinitely until it comes to rest.

(**e**) Show that, according to this model, the total vertical distance travelled by the ball until it comes to rest is $9.43\,\text{m}$, correct to three significant figures. [4]

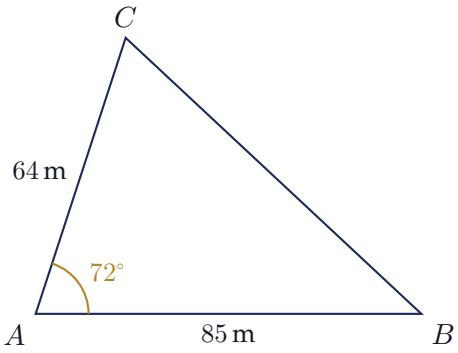
(**f**) Explain why this model is unlikely to describe the behaviour of a real ball exactly. [3]

Turn over ▶

4. [Maximum mark: 14]

A surveyor measures a triangular field ABC. The side AB has length $85\,\text{m}$, the side AC has length $64\,\text{m}$, and the angle $B\hat{A}C = 72°$.

diagram not to scale



(**a**) Find the length of side BC. [3]

(**b**) Find the size of the angle $A\hat{B}C$. [3]

(**c**) Find the area of the field. [3]

The field is to be drawn on a plan using a scale of $1 : 2000$.

(**d**) Determine the area of the field on the plan, in square centimetres. [3]

(**e**) State one reason why the calculated area may not equal the true surface area of the field. [2]

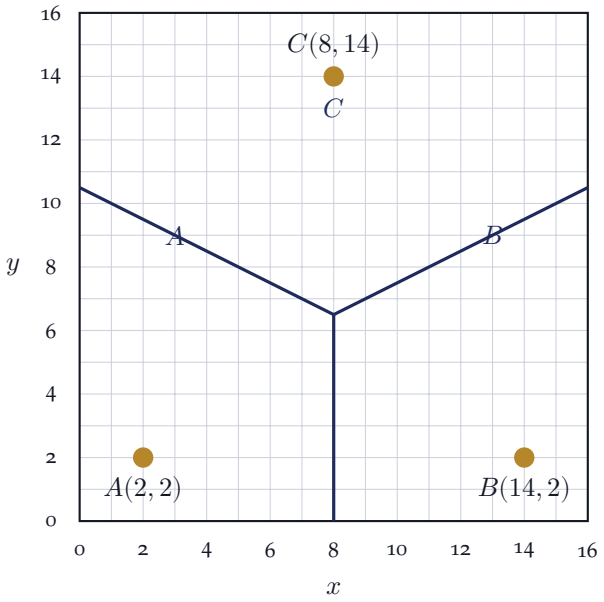
Turn over ▶

5. [Maximum mark: 19]

An island is monitored by three drone stations, A, B and C. On a coordinate grid where 1 unit represents 2 km, the stations are located at

$$A(2,\ 2), \qquad B(14,\ 2), \qquad C(8,\ 14).$$

A Voronoi diagram is used to assign to each station the region of the island closest to it.



(a) (i) Find the coordinates of the midpoint of $[AC]$. [1]

(ii) Find the gradient of the perpendicular bisector of $[AC]$. [2]

(iii) Hence show that the boundary between the regions of stations A and C has equation $y = -0.5x + 10.5$. [1]

An incident is reported at the point $Z(11,\ 5)$.

(b) (i) Using the Voronoi diagram, identify which station is expected to respond to the incident at Z, and justify your answer. [2]

(ii) Suggest one reason why a different station might respond instead. [1]

The boundary between the regions of stations A and B is the line $x = 8$.

(c) A fourth station is to be placed at the point D that is equidistant from all three existing stations. Find the coordinates of D. [3]

(d) Find the actual distance, in kilometres, from the new station D to station A. [2]

This question continues on the following page.

Turn over ▶

(Question 5 continued)

After a report is received, a drone is launched from station A. For the first 20 seconds of flight, its speed v, in metres per second, is modelled by

$$v(t) = 12 + 0.8t - 0.03t^2, \qquad 0 \le t \le 20,$$

where t is the time in seconds after launch.

(e) Find the distance travelled by the drone during the first 20 seconds of flight. [4]

(f) Explain one reason why the straight-line distance from a station to an incident may not determine which drone actually reaches the incident first. [3]

End of Mock Examination 3

Total: 80 marks