IB DEMYSTIFIED
EXAMINERS · MODERATORS · AUTHORS

IB MATHEMATICS · FORMULA BOOKLET

Mathematics: Analysis & Approaches

Standard and Higher Level formula booklet, organised by syllabus topic

SL HL

www.ibdemystified.com

JUMP TO Prior learning · Number and algebra · Functions · Geometry and trigonometry · Statistics and probability · Calculus

ASSUMED KNOWLEDGE

Prior learning

Area of a parallelogram	$A = bh$ b = base length, h = perpendicular height
Area of a triangle	$A = \frac{1}{2}bh$ b = base length, h = perpendicular height
Area of a trapezium (trapezoid)	$A = \frac{1}{2}(a+b)\,h$ a, b = lengths of the two parallel sides, h = perpendicular distance between them
Area of a circle	$A = \pi r^2$ r = radius
Circumference of a circle	$C = 2\pi r$ r = radius
Volume of a cuboid	$V = lwh$ l = length, w = width, h = height
Volume of a cylinder	$V = \pi r^2 h$ r = radius, h = height

Volume of a prism	$V = Ah$ A = area of the uniform cross-section, h = length (height)
Curved surface area of a cylinder	$A = 2\pi rh$ r = radius, h = height
Distance between two points in a plane	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for the points (x_1, y_1) and (x_2, y_2)
Midpoint of a line segment (2D)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ for the end points (x_1, y_1) and (x_2, y_2)

TOPIC 1

Number and algebra

1.2 **Arithmetic sequence: nth term**	$u_n = u_1 + (n-1)d$ u_1 = first term, d = common difference
1.2 **Arithmetic series: sum of the first n terms**	$S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right) \qquad S_n = \frac{n}{2}\left(u_1 + u_n\right)$
1.3 **Geometric sequence: nth term**	$u_n = u_1 r^{\,n-1}$ r = common ratio
1.3 **Geometric series: sum of the first n terms**	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r} \qquad r \neq 1$
1.4 **Compound interest**	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ FV = future value, PV = present value, n = number of years, k = number of compounding periods per year, $r\%$ = nominal annual interest rate
1.5 **Link between exponents and logarithms**	$a^x = b \iff x = \log_a b \qquad a, b > 0,\ a \neq 1$
1.7 **Laws of logarithms**	$\log_a xy = \log_a x + \log_a y \qquad \log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$
1.7 **Change of base**	$\log_a x = \frac{\log_b x}{\log_b a}$
1.7 **Exponential and logarithmic functions**	$a^x = \mathrm{e}^{x \ln a} \qquad \log_a a^x = x = a^{\log_a x}$ where $a, x > 0$ and $a \neq 1$
1.8 **Sum to infinity of a geometric series**	$S_\infty = \frac{u_1}{1 - r} \qquad \lvert r \rvert < 1$
1.9 **Binomial theorem, $n \in \mathbb{N}$**	$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + \cdots + {}^n\mathrm{C}_r\, a^{n-r}b^r + \cdots + b^n$
1.9 **Binomial coefficient**	${}^n\mathrm{C}_r = \frac{n!}{r!\,(n-r)!}$

1.10 HL **Combinations and permutations**	$${}^{n}\mathrm{C}_{r} = \frac{n!}{r!\,(n-r)!} \qquad {}^{n}\mathrm{P}_{r} = \frac{n!}{(n-r)!}$$
1.10 HL **Binomial expansion for rational n**	$$(a+b)^{n} = a^{n}\left(1 + n\left(\frac{b}{a}\right) + \frac{n(n-1)}{2!}\left(\frac{b}{a}\right)^{2} + \cdots\right)$$ $n \in \mathbb{Q}$; valid for $\left\lvert\frac{b}{a}\right\rvert < 1$
1.12 HL **Complex number in Cartesian form**	$$z = a + b\mathrm{i}$$
1.13 HL **Modulus–argument (polar) and Euler forms**	$$z = r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{e}^{\mathrm{i}\theta} = r\,\mathrm{cis}\,\theta$$ $r = \lvert z\rvert$, $\theta = \arg z$
1.14 HL **De Moivre's theorem**	$$[r(\cos\theta + \mathrm{i}\sin\theta)]^{n} = r^{n}(\cos n\theta + \mathrm{i}\sin n\theta) = r^{n}\mathrm{e}^{\mathrm{i}n\theta} = r^{n}\,\mathrm{cis}\,n\theta$$

TOPIC 2

Functions

2.1 **Forms of the equation of a straight line**	$y = mx + c \qquad ax + by + d = 0 \qquad y - y_1 = m(x - x_1)$
2.1 **Gradient through two points**	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
2.6 **Axis of symmetry of a quadratic graph**	$f(x) = ax^2 + bx + c \Rightarrow \text{axis of symmetry: } x = -\dfrac{b}{2a}$
2.7 **Roots of a quadratic equation**	$ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a \neq 0$
2.7 **Discriminant**	$\Delta = b^2 - 4ac$
2.12 HL **Sum and product of the roots of a polynomial**	$\sum_{r=0}^{n} a_r x^r = 0 \Rightarrow \text{sum} = \dfrac{-a_{n-1}}{a_n} \qquad \text{product} = \dfrac{(-1)^n a_0}{a_n}$

TOPIC 3

Geometry and trigonometry

3.1 **Distance between two points in space**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ for the points (x_1, y_1, z_1) and (x_2, y_2, z_2)
3.1 **Midpoint of a line segment (3D)**	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
3.1 **Volume of a right pyramid**	$V = \frac{1}{3}Ah$ A = area of the base, h = vertical height
3.1 **Volume of a right cone**	$V = \frac{1}{3}\pi r^2 h$ r = base radius, h = vertical height
3.1 **Curved surface area of a cone**	$A = \pi r l$ r = base radius, l = slant height
3.1 **Volume of a sphere**	$V = \frac{4}{3}\pi r^3$ r = radius
3.1 **Surface area of a sphere**	$A = 4\pi r^2$ r = radius
3.2 **Sine rule**	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ side a is opposite angle A, and so on
3.2 **Cosine rule**	$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$
3.2 **Area of a triangle (two sides and included angle)**	$A = \frac{1}{2}ab\sin C$
3.4 **Arc length (angle in radians)**	$l = r\theta$ r = radius, θ = angle at the centre in radians
3.4 **Area of a sector (angle in radians)**	$A = \frac{1}{2}r^2\theta$ r = radius, θ = angle at the centre in radians

3.5 **Tangent identity**	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
3.6 **Pythagorean identity**	$\cos^2\theta + \sin^2\theta = 1$
3.6 **Double angle identities**	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
HL 3.9 **Reciprocal trigonometric ratios**	$\sec\theta = \dfrac{1}{\cos\theta} \qquad \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$
HL 3.9 **Further Pythagorean identities**	$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$
HL 3.10 **Compound angle identities**	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
HL 3.10 **Double angle identity for tangent**	$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$
HL 3.12 **Magnitude of a vector**	$\lvert\boldsymbol{v}\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$ for $\boldsymbol{v} = (v_1, v_2, v_3)$
HL 3.13 **Scalar (dot) product**	$\boldsymbol{v}\cdot\boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3 \qquad \boldsymbol{v}\cdot\boldsymbol{w} = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$ θ = angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
HL 3.13 **Angle between two vectors**	$\cos\theta = \dfrac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert}$
HL 3.14 **Equations of a line in 3D**	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} \qquad x = x_0 + \lambda l,\ y = y_0 + \lambda m,\ z = z_0 + \lambda n$ $\dfrac{x - x_0}{l} = \dfrac{y - y_0}{m} = \dfrac{z - z_0}{n}$ vector, parametric and Cartesian forms; direction vector $\boldsymbol{b} = (l, m, n)$
HL 3.16 **Vector (cross) product**	$\boldsymbol{v}\times\boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix} \qquad \lvert\boldsymbol{v}\times\boldsymbol{w}\rvert = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\sin\theta$

3.16 HL **Area of a parallelogram from vectors**	$A = \lvert \boldsymbol{v} \times \boldsymbol{w} \rvert$ $\boldsymbol{v}$ and $\boldsymbol{w}$ form two adjacent sides
3.17 HL **Equations of a plane**	$\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b} + \mu \boldsymbol{c}$ $\quad \boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$ $\quad ax + by + cz = d$ vector form, normal form (normal vector $\boldsymbol{n}$) and Cartesian form

TOPIC 4

Statistics and probability

4.2 **Interquartile range**	$\mathrm{IQR} = Q_3 - Q_1$
4.3 **Mean of a data set (frequency form)**	$\bar{x} = \dfrac{\sum_{i=1}^{k} f_i x_i}{n} \qquad n = \sum_{i=1}^{k} f_i$ f_i = frequency of the value x_i
4.5 **Probability of an event A**	$\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$ $n(U)$ = number of outcomes in the sample space
4.5 **Complementary events**	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
4.6 **Combined events (addition rule)**	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
4.6 **Mutually exclusive events**	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
4.6 **Conditional probability**	$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
4.6 **Independent events**	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$
4.7 **Expected value of a discrete random variable X**	$\mathrm{E}(X) = \sum x\,\mathrm{P}(X = x)$
4.8 **Binomial distribution: mean and variance**	$X \sim \mathrm{B}(n, p) \qquad \mathrm{E}(X) = np \qquad \mathrm{Var}(X) = np(1-p)$
4.12 **Standardised normal variable**	$z = \dfrac{x - \mu}{\sigma}$ μ = mean, σ = standard deviation
HL 4.13 **Bayes' theorem**	$\mathrm{P}(B \mid A) = \dfrac{\mathrm{P}(B)\mathrm{P}(A \mid B)}{\mathrm{P}(B)\mathrm{P}(A \mid B) + \mathrm{P}(B')\mathrm{P}(A \mid B')}$ $\mathrm{P}(B_i \mid A) = \dfrac{\mathrm{P}(B_i)\mathrm{P}(A \mid B_i)}{\mathrm{P}(B_1)\mathrm{P}(A \mid B_1) + \mathrm{P}(B_2)\mathrm{P}(A \mid B_2) + \mathrm{P}(B_3)\mathrm{P}(A \mid B_3)}$

4.14 HL **Variance and standard deviation of a data set**	$\sigma^2 = \dfrac{\sum\limits_{i=1}^{k} f_i(x_i - \mu)^2}{n} = \dfrac{\sum\limits_{i=1}^{k} f_i x_i^2}{n} - \mu^2 \qquad \sigma = \sqrt{\dfrac{\sum\limits_{i=1}^{k} f_i(x_i - \mu)^2}{n}}$
4.14 HL **Linear transformation of a random variable**	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b \qquad \mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$
4.14 HL **Expected value of a continuous random variable**	$\mathrm{E}(X) = \mu = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x$
4.14 HL **Variance**	$\mathrm{Var}(X) = \mathrm{E}\left[(X - \mu)^2\right] = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$
4.14 HL **Variance of a discrete random variable**	$\mathrm{Var}(X) = \sum (x - \mu)^2 \,\mathrm{P}(X = x) = \sum x^2 \,\mathrm{P}(X = x) - \mu^2$
4.14 HL **Variance of a continuous random variable**	$\mathrm{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,\mathrm{d}x = \int_{-\infty}^{\infty} x^2 f(x)\,\mathrm{d}x - \mu^2$

TOPIC 5

Calculus

5.3 **Derivative of x^n**	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
5.12 HL **Derivative from first principles**	$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
5.5 **Integral of x^n**	$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C \qquad n \neq -1$
5.5 **Area between a curve and the x-axis, where $f(x) > 0$**	$A = \int_a^b y \, \mathrm{d}x$
5.6 **Standard derivatives**	$f(x) = \sin x \Rightarrow f'(x) = \cos x \qquad f(x) = \cos x \Rightarrow f'(x) = -\sin x$ $f(x) = \mathrm{e}^x \Rightarrow f'(x) = \mathrm{e}^x \qquad f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
5.6 **Chain rule**	$y = g(u),\ u = f(x) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$
5.6 **Product rule**	$y = uv \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$
5.6 **Quotient rule**	$y = \frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
5.15 HL **Further standard derivatives**	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x \qquad f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$ $f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$ $f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x \qquad f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$ $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$ $f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$ $f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$ $f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$

5.9 **Acceleration**	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2}$
5.9 **Distance travelled from t_1 to t_2**	$\text{distance} = \displaystyle\int_{t_1}^{t_2} \lvert v(t) \rvert \, \mathrm{d}t$
5.9 **Displacement from t_1 to t_2**	$\text{displacement} = \displaystyle\int_{t_1}^{t_2} v(t) \, \mathrm{d}t$
5.10 **Standard integrals**	$\displaystyle\int \frac{1}{x} \, \mathrm{d}x = \ln \lvert x \rvert + C \qquad \int \sin x \, \mathrm{d}x = -\cos x + C$ $\displaystyle\int \cos x \, \mathrm{d}x = \sin x + C \qquad \int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + C$
HL 5.15 **Further standard integrals**	$\displaystyle\int a^x \, \mathrm{d}x = \frac{1}{\ln a} a^x + C \qquad \int \frac{1}{a^2 + x^2} \, \mathrm{d}x = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$ $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}} \, \mathrm{d}x = \arcsin\left(\frac{x}{a}\right) + C, \ \lvert x \rvert < a$
HL 5.16 **Integration by parts**	$\displaystyle\int u \frac{\mathrm{d}v}{\mathrm{d}x} \, \mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x} \, \mathrm{d}x$
5.11 **Area of the region enclosed by a curve and the x-axis**	$A = \displaystyle\int_a^b \lvert y \rvert \, \mathrm{d}x$
HL 5.17 **Area of the region enclosed by a curve and the y-axis**	$A = \displaystyle\int_a^b \lvert x \rvert \, \mathrm{d}y$
HL 5.17 **Volume of revolution about the x- or y-axis**	$V = \displaystyle\int_a^b \pi y^2 \, \mathrm{d}x \qquad V = \int_a^b \pi x^2 \, \mathrm{d}y$
HL 5.18 **Euler's method**	$y_{n+1} = y_n + h \times f(x_n, y_n) \qquad x_{n+1} = x_n + h$ h = constant step length
HL 5.18 **Integrating factor for $y' + P(x)y = Q(x)$**	$\mathrm{e}^{\int P(x) \, \mathrm{d}x}$
HL 5.19 **Maclaurin series**	$f(x) = f(0) + x f'(0) + \dfrac{x^2}{2!} f''(0) + \cdots$

HL

5.19

Maclaurin series for special functions

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots \qquad \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$$

© 2026 IB Demystified · Examiners. Moderators. Authors.